Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 1, Number 2; October, 2014 pp. 98-100

http://www.krishisanskriti.org/jbaer.html

To Design Quantum Unitary Gate using Born Scattering

Shakti Sharma[1], Harish Parthasarathy[2], Akash Tayal[3]

[1,3]*Indira Gandhi Delhi Technical University for Women, New Delhi, India*
[2]*Netaji Subhas Institute of Technology, New Delhi, India*

Abstract: **Classical computers are reaching towards their limit in speed and chip size. Present day need is a shift towards more efficient, fast and secured computational system. Quantum computation and Quantum information is a field in which control of a quantum mechanical system plays a role. The basic building block in quantum Information and computation theory is a Quantum Gate. Quantum Gate is a Unitary operator or a Unitary Gate for evolution of a quantum operation and satisfies condition $U^{-1}=U^*$. Quantum Fourier Transform is a quantum algorithm to perform Fourier transform of quantum mechanical amplitudes.[6] The various applications of Quantum Fourier Transform are in the fields of Phase estimation, Order finding problems, Factoring problems, Eigen values of a unitary operator, Hidden subgroup problems etc.[6]**

In this paper, effort has been made to design a Quantum Unitary Gate. To achieve this, an elastic scattering process with a weak potential is analyzed using the Born approximation. The algorithm to design the QFT gate has been applied.

1. INTRODUCTION

The physical state of a particle is represented by a state vector or 'state ket' in a complex vector space. The set of all state vectors of a single particle constitutes a Hilbert space $\hat{\mathcal{H}}$. A finite dimensional Hilbert space is considered as a Quantum System. The Position, momentum, Hamiltonian etc characteristics of a particle are called observables. These observables are given by Hermitian operators.

A quantum operation is an evolution defined by Unitary operator. This unitary operator is called a Quantum Gate. The evolution is obtained by some scattering experiment. Time Evolution operator is a translation which relates two kets. A linear transformation from one Hilbert space(Domain) to another(Range) is Isometric if Inner product remains same. A Linear operator is unitary if and only if it satisfies condition $U^{-1}=U^*$.[1] Unitary operator U is bounded i.e. $\|U\|=1$.[1] Unitary operator on H maps the whole of H onto the whole of H and preserves the norm. i.e. D (U) = R (U) = H and $\|U\psi\|=\|\psi\|$ for all $|\psi\rangle$. [1] The Inverse of a unitary operator is also defined on the whole of H.

2. SCATTERING THEORY AND BORN APPROXIMATION

A particle with mass μ and momentum p=ħk, at time t_0, is in the state $|\psi_i\rangle$ which is an eigen state of the free particle of Hamiltonian H_0. After passing through a region of potential V, it gets scattered to a state $|\psi_s\rangle$ and finally is in the eigen state of the perturbed Hamiltonian $\hat{H}$.

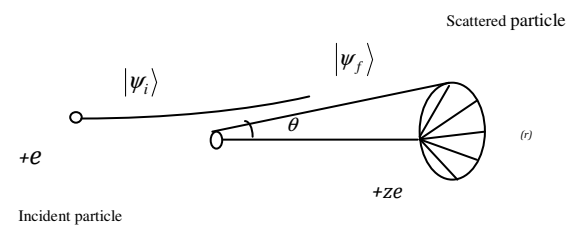

Fig. 1 Evolution using Scattering process

The $|\psi\rangle$ at t=0 is related to the in asymptote $|\psi_{in}\rangle$ and the out asymptote $|\psi_{out}\rangle$. $H_0 = \hbar^2+k^2/2\mu V$ is free Hamiltonian where $k= (2\mu E/ \hbar^2)^{0.5}$. E is the kinetic energy. Interaction between both particles is denoted by $r = r - r'$ and $\alpha = \angle(r, r')$. For $r >> r'$

$$\left|r - r'\right| = \left(r^2 - 2rr'\cos\alpha + r'^2\right)^{1/2} \approx r - \hat{r}\cdot r'$$

Basic schrodinger equation [2]
$(H_0 + V)|\psi\rangle = E|\psi\rangle$ and its Solution is

$$|\psi\rangle = \frac{1}{E - H_0} V|\psi\rangle + |\phi\rangle \quad ..... \qquad (1)$$

Lippmann Schwinger equation for position basis[2]

$$\langle r \mid \psi\rangle = \langle r \mid \phi\rangle + \int d^3r \left\langle r \left| \frac{1}{E - H_0 + i\varepsilon} \right| r' \right\rangle \langle r' \mid V \mid \psi\rangle \quad ..(2)$$

Kernel of integral equation is defined by Green's function[2]

$$G_{\pm}(\mathrm{r},\mathrm{r}')=(-1/4\pi)\frac{e^{\pm ik|r-r'|}}{|r-r'|} \quad \ldots \tag{3}$$

Due to weak interaction of particle with potential, the effects of multiple scattering can be avoided and a first order approximation of the scattering states will work. Substituting kernel and for larger r, Born approximation expression for scattering is

$$\psi_s = -\frac{1}{4\pi}\frac{2m}{\hbar^2}\frac{e^{ikr}}{r}\int d^3x' e^{-ik'x'}V(x')\psi_i(\mathrm{r}') \quad \ldots. \tag{4}$$

The Born Approximation of the scattering amplitude is the Fourier Transform of the scattering potential.[9] Final wave is a summation of incident and scattered wave. $\psi_f = \psi_i + \psi_s$

$$\psi_f = \psi_i - \frac{1}{4\pi}\frac{2m}{\hbar^2}\frac{e^{ikr}}{r}\int d^3x' e^{-ik'x'}V(x')\psi_i(\mathrm{r}') \tag{5}$$

3. MULTIPOLE MOMENT POTENTIAL

The scattering potential is assumed to be generated by a distribution of charge over a small region B of space. The potential $V(\underline{r})$ then satisfies Poisson's equation:

$$\nabla^2 V(\underline{r}) = -\frac{\rho(\underline{r})}{\varepsilon_0} \quad \ldots. \tag{6}$$

Where $\rho(\underline{r})$ is the charge density. Equivalently the solution of eq (6) is

$$V(\underline{r}) = \int_B \frac{\rho(\underline{r}')}{4\pi\varepsilon_0|\underline{r}-\underline{r}'|}d^3r' \quad \ldots. \tag{7}$$

For r>>r', eq (7) can be approximated by

$$V(\underline{r}) = \sum_{l=0}^{N}\frac{C_l P_l(\cos\theta)}{r^{l+1}} \quad \ldots. \tag{8}$$

Assuming $\rho(r')$ is azimuthally symmetric.

Potential in product form in spherical coordinates is $\Phi = \frac{U(\mathrm{r})}{r}P(\theta)Q(\phi)$ When θ in P(θ) is expressed in terms of x=cos θ then substituting in Laplace equation in spherical coordinates, generalized Legendre equation is given[5] by:

$$\frac{d}{dx}\left[\left(1-x^2\right)\frac{dp}{dx}\right]+\left[l\left(l+1\right)-\frac{m^2}{1-x^2}\right]P=0$$

Solutions of the above equation are Legendre functions. Legendre polynomials using Rodrigues' formula[5]: $P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$.

4. DESIGNING GATE

Quantum scattering is used here to design a finite dimensional quantum unitary gate. When the scattering potential is small, the outstate is a small perturbation of the input state.

$$|\psi_f\rangle = |\psi_i\rangle + \varepsilon T_v|\psi_i\rangle$$
$$|\psi_f\rangle = (I+\varepsilon T_v)|\psi_i\rangle \quad \ldots\ldots. \tag{9}$$

Where Tv is a skew Hermitian operator determined by the potential V. V is selected so that $\|I+\varepsilon Tv-U_d\|$ is a minimum,where Ud is Unitary in nature. The initial state

$$\psi_i(\underline{r}) = C\cdot\exp\left(jk\hat{n}_i\cdot\underline{r}\right) \quad \ldots. \tag{10}$$

where $\hat{n}_i$ are unit vectors.

Using Born scattering, the final state is given by

$$\psi_i^f(\underline{r}) = \frac{\delta m_0 C}{2\pi\hbar^2}\int V(\underline{r}')\psi_i(\underline{r}')\frac{\exp\left(jk|\underline{r}-\underline{r}'|\right)}{|\underline{r}-\underline{r}'|}d^3r' + \psi_i(\underline{r}) \tag{11}$$

V is taken as multipole potential of the form

$$V(r,\theta) = \sum_{l=0}^{N}\frac{c(l)P_l(\cos\theta)}{r^{l+1}} \quad \ldots. \tag{12}$$

Then Tv is of the form

$$Tv = \sum_{l=0}^{N}c(l)V_l(r,\theta) \equiv \sum_{l=0}^{N}c(l)V_l(\underline{r}) \quad \ldots \tag{13}$$

where V_l (r,θ) is completely determined by the projectile moment $\hbar\underline{ki} = \hbar k\hat{ni}$.

The C(l)s are to be selected so that $\sum_{i=1}^{k}\left\|\psi_i^d - \psi_i^f\right\|^2$ is a minimum. where $\psi_i^d = \underline{\underline{U}}_d\psi_i$, $1 \le i \le k$ and $\underline{\underline{U}}_d$ is desired unitary gate.

$$\psi_i^f(r,\theta) = \frac{\dfrac{\delta m_0 C}{2\pi\hbar^2}\sum_{l=0}^{N} C(\mathrm{l})\displaystyle\int \frac{P_l(\cos\theta')\psi_i(\mathrm{r}',\theta')\exp(-\mathrm{jk}(\mathrm{r}^2+r'-2\mathrm{rr}'(\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos\phi'))^{1/2})}{r'^2\sin\theta' dr' d\theta' d\phi'}}{\left(r^2+r'^2-2rr'(\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos\phi')\right)^{1/2}} + \psi_i(\bar{r},\bar{\theta})$$

where $1 \le i \le k$ …… (14)

In other words C(l)'s are chosen so that the average error energy between the desired set of final states and the actual set of final state is a minimum. This is a simple quadratic function optimization problem and the optimal equations form a set of linear equations for {C(l)} that are solved by matrix inversion using MATLAB.

5. RESULTS AND DISCUSSIONS

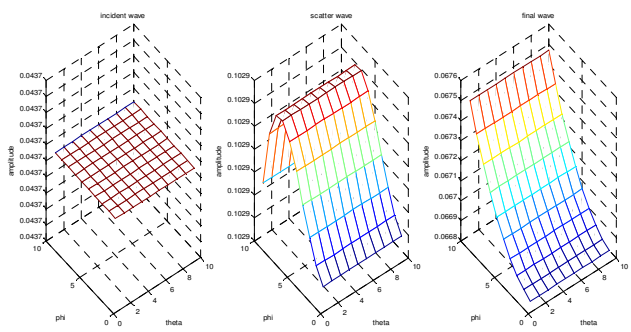

Fig. 2 scattering pattern of Gate

A Matlab implementation has been done to obtain Quantum Unitary Gate. A particle of position vector $\bar{r}(\mathrm{r},\theta,\phi)$ is incident and scattered to vector $\bar{r}'(r',\theta',\phi')$. Here $\phi = 0$ is taken and for incident parameters r,θ and scattered parameters r',θ',ϕ', scattering amplitude is calculated. Software implementation of expressions for incident, scattered and final waves is done. Figure 2 represents example of incident, scattered and final wave patterns.

6. CONCLUSION

An effort has been made in this paper to design a quantum gate using elastic scattering process and Born approximation. Approximation for coefficient of potential is done so that gate can be maximally optimized towards Unitary. For this a small perturbation has been applied to correct evolution operator.

REFERENCES

[1] John Taylor, "Scattering Theory: The Quantum Theory on Nonrelativistic Collisions", John Wiley & Sons, Inc.

[2] J.J.Sakurai,"Modern Quantum Mechanics" Revised Edition, *Addison-Wesley Publishing Company.*

[3] Harish Parthasarathy, "Applications of Advanced Signal Analysis", *I.K International Publishing House Pvt. Ltd.*

[4] Tosio Kato,"Perturbation Theory for Linear Operators", *Springer*

[5] John David Jackson, "Classical Electrodynamics",*Third Edition, John Wiley & Sons, Inc.*

[6] Michael A. Nielsen and Isaac L. Chuang, "Quantum Computation and Quantum Information", *First South Asian Edition 2002, Cambridge University Press*.

[7] Eduard Prugovecki, "Quantum Mechanics in Hilbert space" *Academic Press, New York and London.*

[8] P.A.M. Dirac, "The Principles of Quantum Mechanics" *Fourth Edition, Oxford University Press.*

[9] Leonard I. Schiff, "Quantum Mechanics" *Third Edition, McGRAW- HILL Book Company.*

[10] Adriano Barenco, Charles H. Bennett, Richard Cleve, David P. DiVincenzo, Norman Margolus,Peter Shor,Tycho Sleator, John Smolin,Harald Weinfurter, "Elementary gates for quantum computation", *arxiv: quant-ph/9503016v1 23 Mar 1995.*

[11] Chapter 2 "Quantum Gates", Springer, *Explorations in Quantum Computing, Williams, C.P. 2011, XXII.*

[12] "Introduction to Scattering Theory", *Phy 852, Quantum Mechanics II, Spring 2008*