Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 1, Number 2; October, 2014 pp. 38-42

http://www.krishisanskriti.org/jbaer.html

Application of Fractal Analysis in Pavement Materials

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Abstract: **The Aggregates shape, size and texture are key parameters in which the quality of asphalt concrete depends. The shape of particles affects the durability, strength and optimum binder content. Proper design air void and density can be achieved through measurements of these aggregate parameters. Shape and Texture of aggregates also influence the surface characteristics of pavement and also the cost effectiveness of asphalt concrete. The shape of the aggregates accessed by Euclidean geometry lacks in accuracy due to the irregular shape of aggregates. Albeit, the fractal analysis which uses the concept of fractal dimensions can address the issue of irregular geometry. In this paper basic fractal theory is explained and its applicability in various pavement material problems are discussed.**

Keywords : Fractal, Fractal Analysis, Aggregate Shape etc.

1. INTRODUCTION

The significance of aggregate particle shape on their mechanical properties can be easily acknowledged. In asphalt concrete, the physical characteristics of aggregate particles has been related to durability, workability, shear resistance, tensile strength, hardness, and fatigue behavior [1]. The shape of aggregates practical also affects the overall surface characteristics of the pavement like roughness, road profile etc. The successful measurement of aggregate geometry is all important for determining their effects on pavement performance and for selecting aggregates to produce pavements of adequate quality. The popular approach to access aggregate physical properties is established along the concept of Euclidian geometry which is based on regular shape geometry. Euclidean geometry concerned largely with sets and functions to which the methods of classical calculus can be applied i.e. Sets or functions that are sufficiently smooth or regular. Although the aggregate particle shape is non-smooth and irregular in nature, so the application of Euclidean Theory results in a substantial amount of error. In the Euclidean Theory the geometry of matter is measured on the set of integers, the curves are mapped by the integer 1 (one dimensional), surface by 2 and volume by 3. However the shapes can better represent by values between these 2 integers. Fractal theory provides a general framework for the study of irregular sets and shapes. In Fractal Analysis the geometry of matter is described in the set of real non-integer numbers known as fractal dimension (FD). For object lesson if a one dimensional curve is extremely rough, therefore this condition can be analyzed more efficiently by handling it as surface i.e. making it two dimensional. Hence a fractal dimension lies between two integer dimensions [2]. In his paper the concept of fractal theory is discussed and its applicability to various pavement engineering problems is presented.

2. FRACTAL GEOMETRY

All Irregular geometry provides more respectable representation of natural phenomena like tree, mountains, clouds and rocks etc. Than traditional geometry. Fractal geometry is the branch of mathematics which involves the study of irregular shape geometry. The basic concept of fractal geometry is introduced by Mandelbrot [3], [4] and later it emerges as the powerful tool for researchers to analyze the irregular shape and texture of objects. Fractal is defined in terms for fractal dimensions. The process of seeing out the fractal dimensions of a body is termed as fractal analysis.

The fractals can be understood by examples of middle third Cantor and von Koch curve. The middle third Cantor set is one of the best recognized and most easily constructed shapes which display many fractal characteristics. It can be made by a sequence of deletion operations from a given set. Refer Fig. 1. The set E_1 of the Fig 1 is represented by a line having points between intervals $[0, a]$. The next set E_2 can be obtained by deleting the middle third portion of set E_1 so the new line, lies between $\left[0, \frac{a}{3}\right]$ and $\left[\frac{2a}{3}, a\right]$. Deleting the middle thirds of intervals $\left[0, \frac{a}{3}\right]$ and $\left[\frac{2a}{3}, a\right]$, third set E_3 of Cantor can be obtained which comprise of four intervals $\left[0, \frac{a}{9}\right], \left[\frac{2a}{9}, \frac{a}{3}\right], \left[\frac{2a}{3}, \frac{7a}{9}\right], \left[\frac{8a}{9}, a\right]$ and so on.

In general the set E_k can be obtained by deleting middle third of all the segments contained by set E_{k-1}. Thus set E_k contains 2^k segments of length 3^{-k}.

The Von Koch curve refer Fig 2, obtained from series of removal operations. The set E_k which consists four line segments can be obtained by deleting middle third portions of E_{k-1} and replacing it with two equal lines equals in length of the deleted portion of set E_{k-1}.

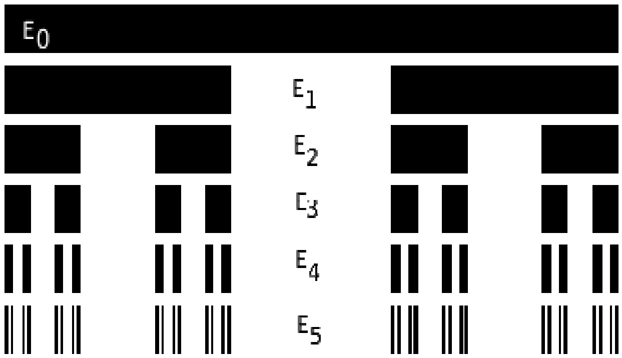


Fig. 1. Construction of the middle third Cantor set

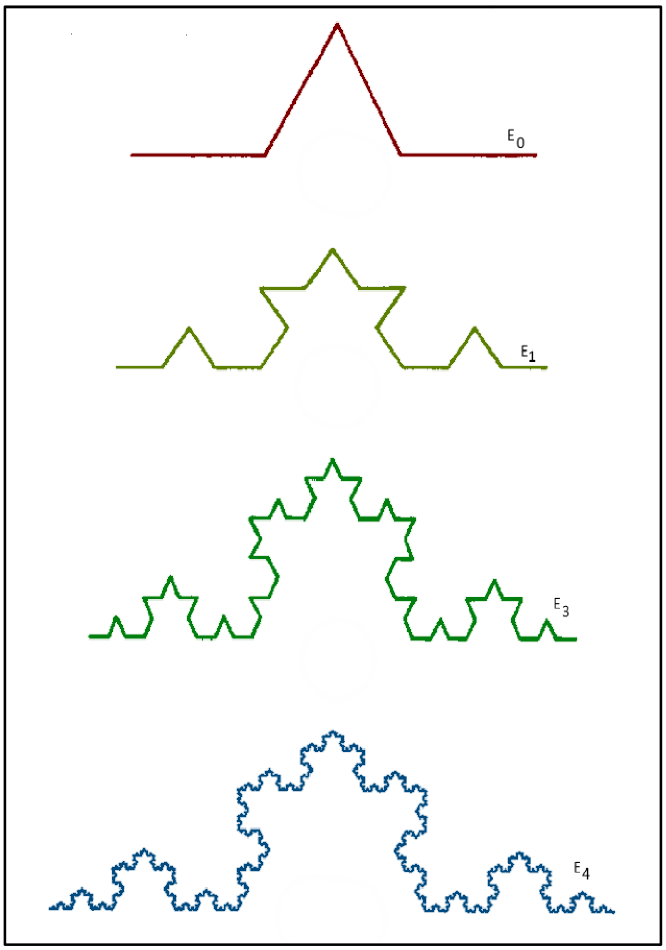


Fig. 2. Construction of the Von Koch curve

- Both these middle third Cantor and Von Koch curve have the following features which resemble their fractal characteristics [5].
- Both curves are self-similar. The right segment of E_1 is similar to left segment of E_1. Scaled by some factor from E_0.
- Both shapes are detailed structures in spite of its simple definition, as the magnification increases the numbers of segment increases.
- The accuracy in calculation of the set increases with the increase in numbers of deletion operations.
- These curves are not the locus of points which explained by classical geometry or by some equation.
- Both curves have irregular geometry.
- These curves cannot be measured in terms of quantities like length.
- A. Fractal Dimension

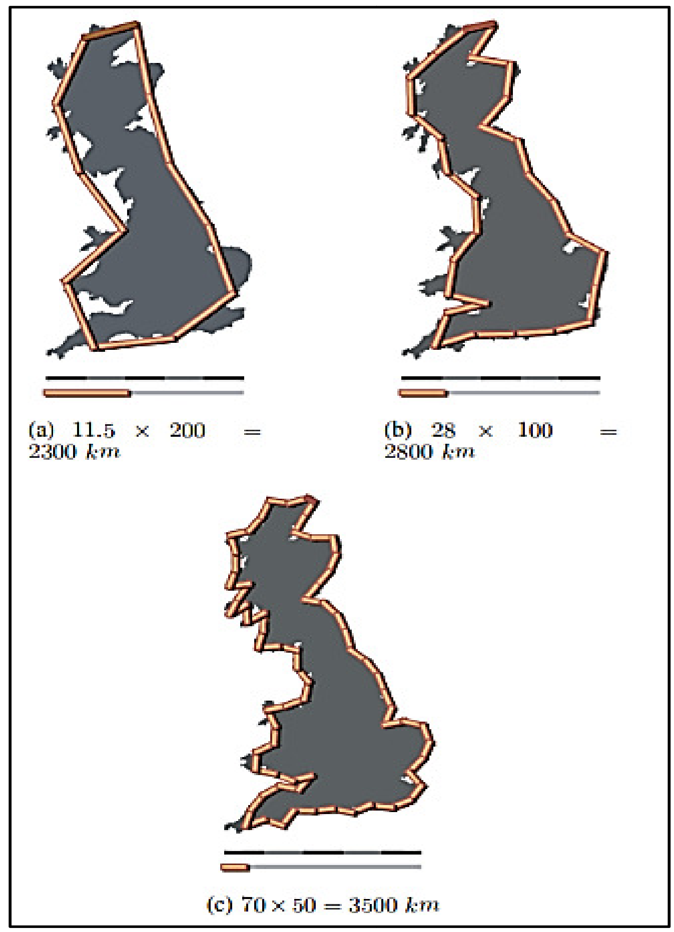


Fig. 3. Effect of length of scale on measurement [8].

3. FRACTAL DIMENSION

A fractal dimension is an index which compares the change in measured quantity with regard to change in measuring scale [6], [7]. Fractal dimension indicates the amount of space occupied near to each point of the set. The length of the coastline measured (refer Fig. 3) is dependent on the length of measuring stick. If the measuring scale used is large the measured length will be less and it will increase with decrease in length of measuring stick.

A given set or shape can be measured using the scale δ but it will ignore all the irregularities smaller than δ. The effect of scale for the measurement can be obtained as $\delta \rightarrow 0$. For example, if F is the curve, then the length measured by scale δ in N_n numbers of linear steps, will be be given as

$$M_\delta(F) = N_n \times \delta \quad (1)$$

This holds good as per the classical geometry. Here the measured value $M_\delta(F)$ is independent of length of scale δ. Although $M_\delta(F)$ holds power law if $\delta \rightarrow 0$. This can be represented as

$$M_\delta(F) \propto \delta^{-s} \Rightarrow M_\delta(F) \text{ ' } c \times \delta^{-s} \quad (2)$$

Where F has divider dimension s with c regarded as s dimensional length of F. s can also be explained as the fractal dimension increment or decrement from the Euclidean dimension i.e. if α is the fractal dimension the s can be expressed as

$$s = \alpha - 1 \quad (3)$$

Taking both side logarithm of Equation 2

$$\log M_\delta(F) = \log c - s \times \log\delta \quad (4)$$

if $\delta \rightarrow 0$ the value of s can be obtained as

$$s = \lim_{\delta \to 0} \frac{\log M_\delta(F)}{-\log \delta} \quad (5)$$

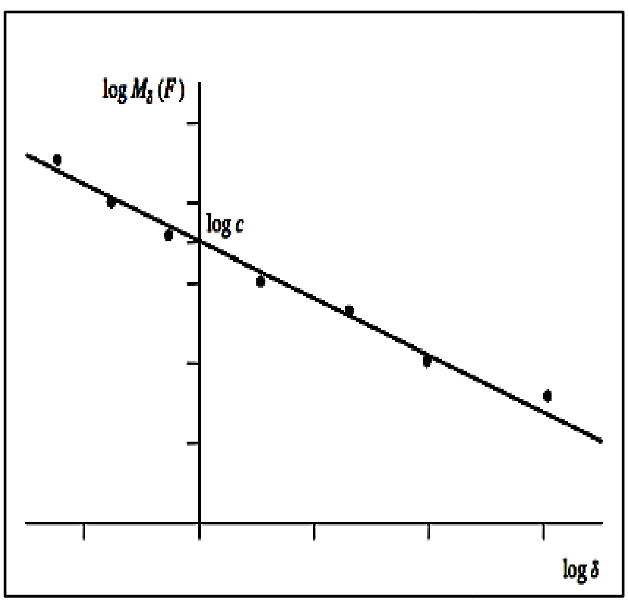


Fig. 4. Empirical estimation of a fractal dimension

Hence the value of s can be obtained by plotting the Equation 2 in log-log plot as shown in Fig.4.

According to Euclidean theory if a curve / area / volume is magnified by scale factor λ then the resulting shape will be multiple of λ / λ^2 / λ^3 respectively. i.e.

$$M(\lambda.A) = \lambda^k.M(A) \quad (6)$$

Where A is the taken shape and k is Euclidean dimension i.e. A is length, area or volume and $k = 1,2,3$. In the case of fractals, shape is having a fractal dimension as α. So the reconsulting change due to λ magnification will be given as

$$M^\alpha(\lambda.A) = \lambda^\alpha.M^\alpha(A) \quad (7)$$

By using the Equation 2 or 7. For example, fractal dimension for the shape shown in Fig. 1 can be estimated as

$$M^\alpha(\lambda.A) = \lambda^\alpha M^\alpha(A) \quad (8)$$

Where λ is magnification factor $\left(\lambda = \frac{1}{3}\right)$. Although the $M^\alpha(A)$ is self-similar, hence Equation 8 reduces to

$$M^\alpha(A) = 2 \times \left(\frac{1}{3}\right)^\alpha M^\alpha(A) \Rightarrow \alpha = \frac{\log 2}{\log 3} = 0.631 \quad (9)$$

In this case the dimensional decrement of 0.369 is due to deletion of segments will the shape less dense. Similarly, fractal dimension for the shape shown in Fig. 2 will be given as

$$\alpha = \frac{\log 4}{\log 3} = 1.262 \quad (10)$$

The dimensional increment of 0.262 is due to intensification of shape because of introduction of two segment in the place of one. There are many other fractal analysis approaches are available like parallel-line, area perimeter, divider etc. The selection of particular approach depends on the problems.

4. APPLICATION OF FRACTAL ANALYSIS

The fractal analysis can be used as a potent instrument in the field of irregular geometry. This tool can also be used in the area of pavement engineering. The pavement engineers are always interested to evaluate the shape characteristics of diverse objects like aggregates, pavement surface and surface cracks etc. The probable uses of fractal analysis in this field are really huge. Some key application of FRA is discussed in this segment.

The shape of an aggregate can be characterized using fractal analysis. The fractal dimension of aggregates can be easily evaluated using area perimeter methods. The proportion of linear extents of a fractal is again a fractal. The roughness fractal dimension (D_R) of the aggregate can be expressed in term of its perimeter (P) and area (A), as [9]

$$c = \frac{P^{D_R}}{A^{0.5}} \tag{11}$$

Where c is constant for similar fractals. Taking Logarithm, Equation 11 cab be reduced as

$$D_R = 2 \times \frac{\log P}{-\log A} = \frac{2}{m} \tag{12}$$

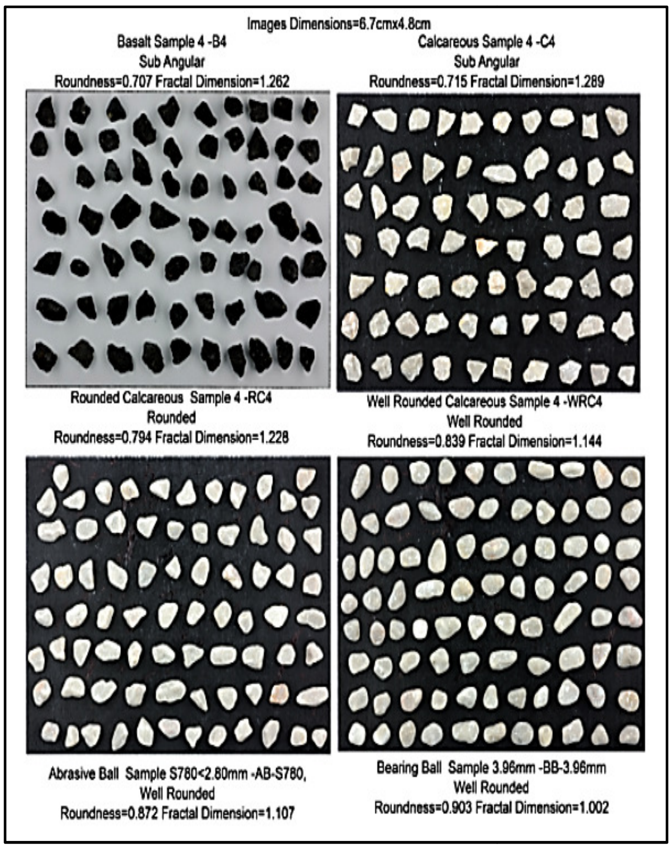


Fig. 5. Photographs used for image processing [9].

Where m is slope coefficient obtained by plotting area and perimeter on a log–log scale. The area and perimeter of the aggregates can be measured by means image processing using any software package or image processing algorithms. Using these measured areas and parameter the fractal dimension can be calculated. Arasan et al. carried on the fractal analysis in order to find out the relation between the fractal dimension and other configuration properties of aggregates using aggregates of several characters. In the study the image of aggregates are processed with ImageJ®, (Refer Fig.5) and a clear relationship between D_R and other aggregate properties are reported. From the Fig. 6, It can be inferred that the aggregate sphericity, roughness and convexity decreases with increase in fractal dimensions although angularity increases. This is because with increases in fractal dimension aggregate becomes more irregular and rough due to the densification of sets near each point. The fractal dimension of the aggregates can also be linked to the mechanical properties of asphalt concrete. The Marshall Stability increases with increase in fractal dimension, although the flow value decreases [1]. Refer Fig. 7

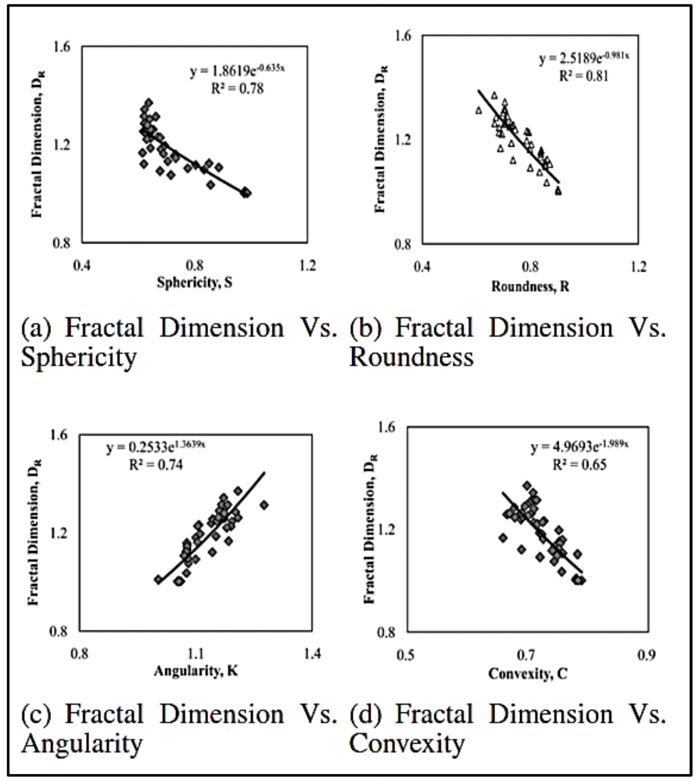


Fig. 6. FD vs. Aggregate properties [9].

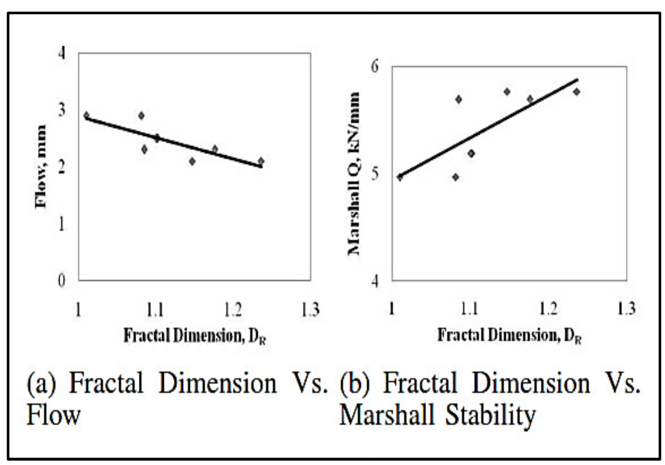
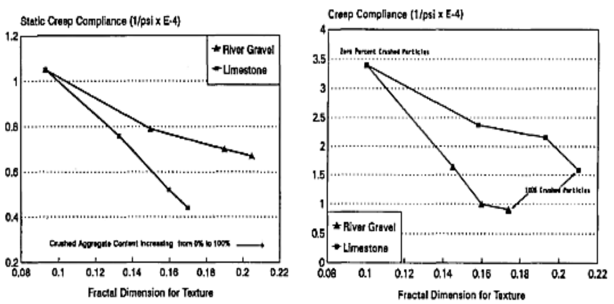


Fig. 7. FD vs. Mechanical Properties [1]



(a) Static Creep Vs. Fractal Dimension (b) Dynamic Creep Vs. Fractal Dimension Stability

Fig. 8. Creep Vs. FD [10]

The creep compliance of the asphalt is also closely related to the fractal dimension of the constituent aggregates. The static and dynamic creep compliance decreases with increase in fractal dimension [10], [11]. Refer Fig.8.

As the fractal dimension (FD) of the aggregates, influences the mix mechanical properties and functioning. Therefore, this fractal dimension of the aggregate can be applied as the standard for selection of aggregates. The compressive strength of asphalt concrete made from the aggregates of same gradation but varying fractal dimension are increases with increase in fractal dimension [12]. Refer Fig. 9. By imposing the fractal criteria better interlocking and density can be attained.

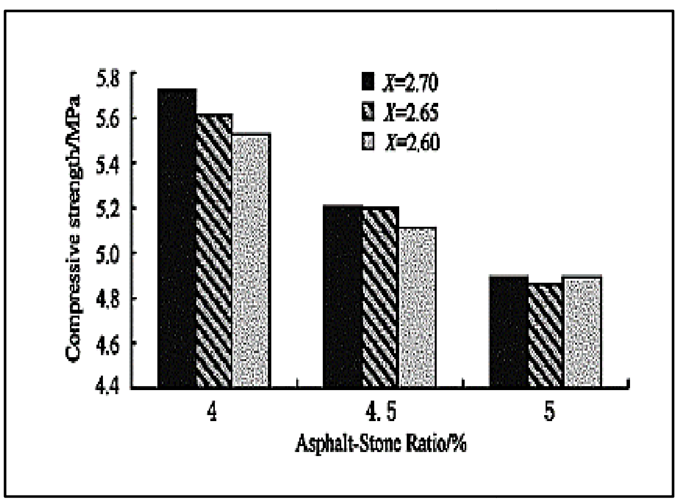


Fig. 9. Compressive Strength VS FD [12]

5. CONCLUSION

The fractal analysis is quite new concept of geometry. It provides an easy and accurate means for the measurement of irregular shapes. Application of fractal analysis in this domain will not only improves the mix performance, but also improves the quality control. Apart from the aggregate characterization this tool can also be used in the study of pavement surface characteristics like surface texture, surface roughness, surface profiling and track geometry. The fractal concept can also be used for modeling of cracks in the fatigue study of asphalt mixes. Much research has been conducted and going along to explore the applicability of this concept in diverse disciplines. Although, more momentum is needed in order to get this concept operational in the arena of pavement engineering. The measurement part of the fractal analysis depends on the algorithms of image processing and imaging techniques. The more study in the field of image processing can make is concept more precise and popular.

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