Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 1, Number 2; October, 2014 pp. 88-93

http://www.krishisanskriti.org/jbaer.html

Analysis of Internally Pressurised Thick Walled Cylinders

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Abstract: **Thick cylinders are used in many industries for various applications which deal with high internal pressures. Hence there is chance of crack initiation inside the cylinder. Stress concentrates in the vicinity of these cracks and hence it is very relevant in analysing the Stress Concentration Factor (SCF). Thick cylinder fails when the pressure greater than the bearing pressure is reached. But in many industries thick cylinders may be subjected to high pressures due to various applications. Autofrettaged cylinder is a best solution to this problem where the bearing capacity of thick cylinders can be improved to a great extent by decreasing the Hoop stress at the vicinity if the inner wall. In the present work SIF at various crack orientations are determined and predicts that which crack fails faster. Also for same wall thickness the SIF for compound cylinder and non-compounded one are compared. The improvement in stresses due to the autofrettaging of cylinders is also analysed. All the analysis are done by FEA (using ANSYS) and analytically. The results are represented as graphs.**

Keyworde: SCF, Autofrettage, FEA, Crack

1. INTRODUCTION

Thick walled cylinders are those which has the thickness to internal diameter ratio more than $1/_{20}$. In the current industrial scenario, these cylinders have a lot of applications mainly in the transportation of materials.

Large internal pressures in thick cylinders produce high tension hoop stresses along the inner surface of the cylinder; the latter may result in the nucleation of the internal surface cracks due to cyclic action of high-pressure loads. Cracks initiate at the surface of the bore, grow into longitudinal or radial cracks and reduce the strength of the structure resulting in premature failure at pressures which are even lower than the design capacity. Thus it is very relevant to study on the behaviour of thick cylinders under high internal pressures. This paper deals with the analysis (both theoretical and FEA)of a thick walled cylinder with and without crack and determining the stress concentration factor. Firstly a benchmark problem is analysed to determine the discrepancy of results obtained theoretically and analytically. The variation in critical crack length with internal pressure is also analysed. Stress intensity factors of cracks at various orientations are determined. Makulsawatdom et al. [3] presented elastic stress concentration factors for internally pressurized thick walled cylinders with radial & offset circular & elliptical cross holes.

Autofrettage is a phenomenon in which thick cylinders are subjected to enormous pressure building in compressive residual stresses. This increases ductile metal's resistance to stress cracking. In case of Autofrettage, material attains state of elastic-plastic state [8]. At particular radius (critical radius) there exists a junction of elasticity and plasticity and is of great importance in designing [9].Hojjati &Hossaini [4] studied the optimum auto frottage pressure & optimum radius of the elastic-plastic boundary of strain-hardening cylinders in plane strain & plane strain conditions. They used both theoretical & finite element (FE) modelling. Equivalent von-Misses stress is used as yield criterion autofrettage of thick cylinders were done analytically using several equations from the supporting literatures the autofrettage radius (critical radius) i.e., the radius separating the plastic and elastic region is determined. Then the pressure applied to convert the inner part of cylinder to plastic state is calculated. The residual stresses are also calculated which is formed both at the plastic and elastic region. Then total stresses are also calculated and the results are graphically represented.

2. BENCHMARK PROBLEM

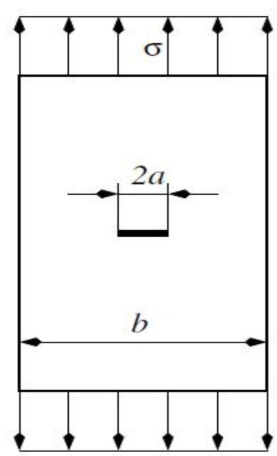
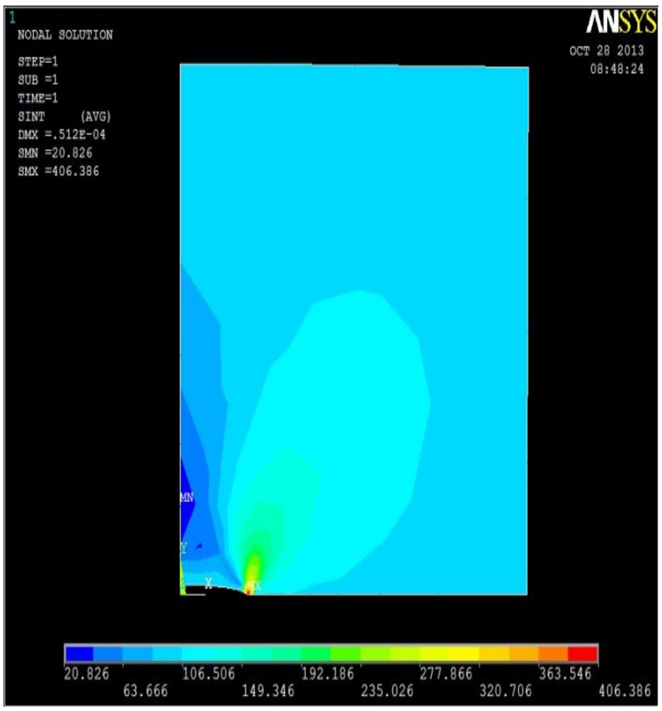


Fig. 2. Steel plate with crack

The benchmark problem is solved to determine the stress intensity factor (K) by using both finite element analysis and analytical equations to find the discrepancy of the results. The material analysed is Mild Steel plate of width 0.2 m having a central crack of length 0.02 m. The material is analysed using ANSYS assuming it to be in isotropic plane strain condition by meshing using 8 node (plane 82) quadrilateral elements. The quarter model is only analysed since the plate is in symmetric geometry. Then the result is validated using analytical equations.



Analytical solution is

Obtained from:

$$K = \sigma F \sqrt{\pi a}$$

$$F = \frac{F'}{\sqrt{(1 - \frac{a}{W})}}$$

Where,
a=half length of the crack
W=half width of the plate
F, F' = function of a, W,H
F' can be obtained from Fig 3.
σ = Pressure applied

The results obtained from FEA and analytical equations are:
From ANSYS; K = 26.34
From theoretical equations; K = 26.53
The discrepancy of two solutions; = 0.07%

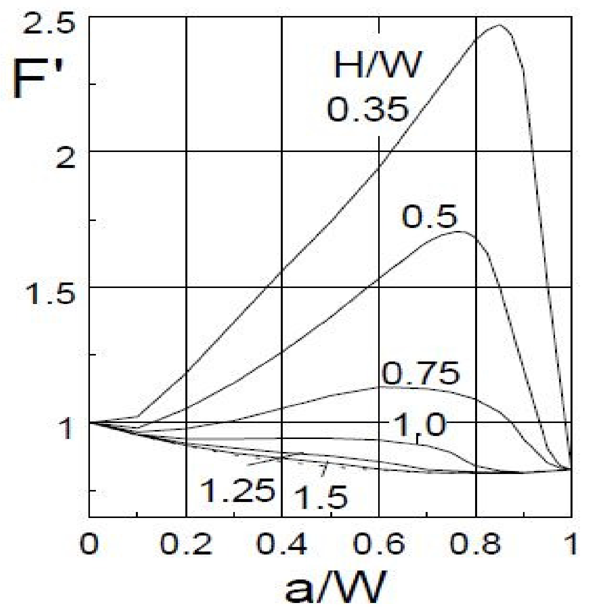


Fig. 3. F' vs a/W

3. BASIC ANALYSIS OF THICK WALLED CYLINDERS

A two dimensional model of internally pressurised thick walled cylinder was modelled for different radius ratios. Radial stresses and Hoop stresses for different radius ratios are determined under different pressure ratios. The following results were obtained:

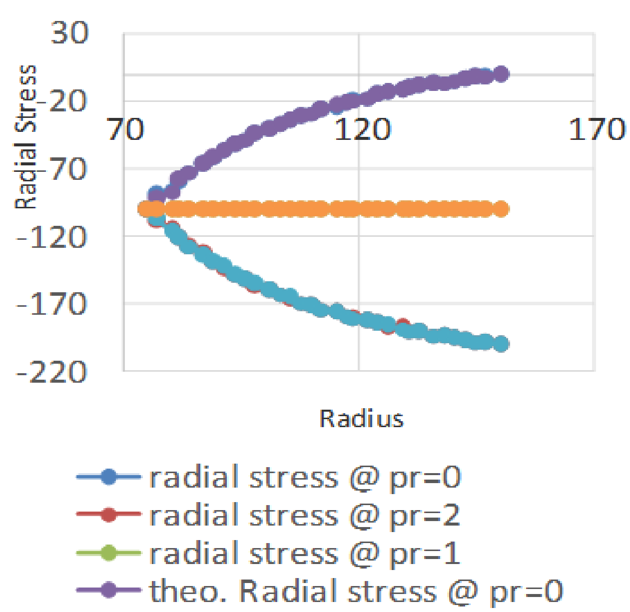


Fig. 4. Radial stress vs wall thickness @ ro/ri=2

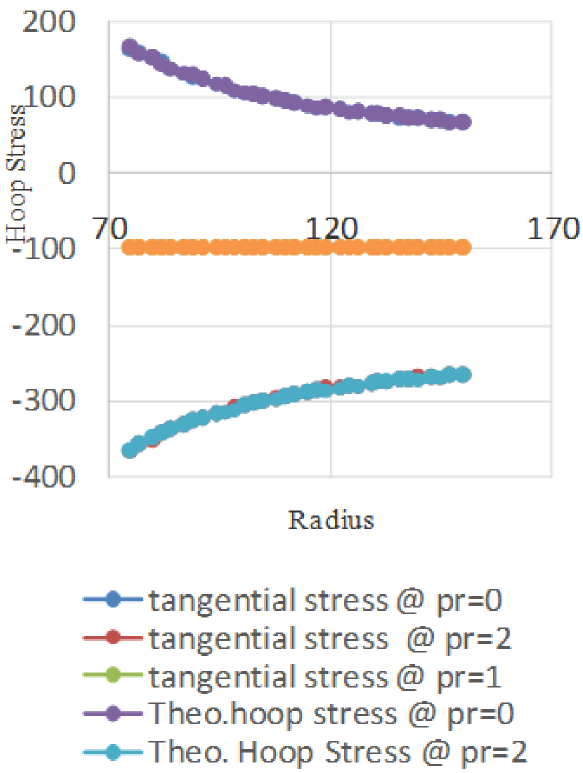


Fig. 5 Hoop stress vs wall thickness @ ro/ri=2

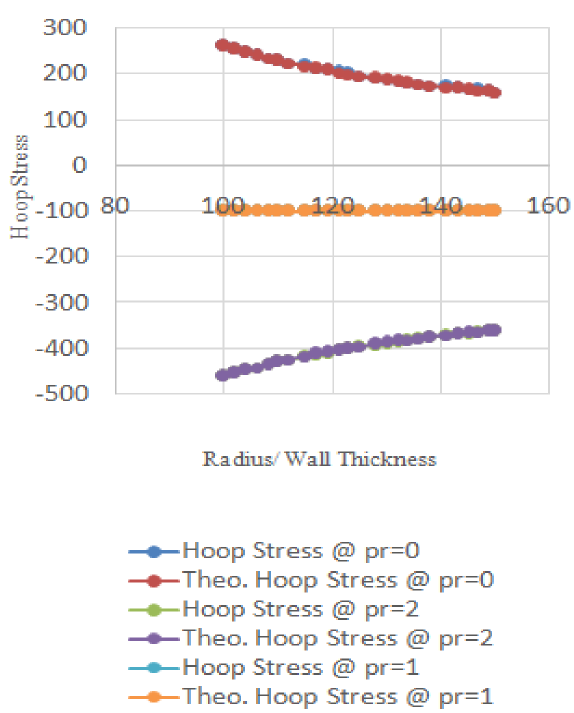


Fig. 6 Hoop stress vs wall thickness @ ro/ri=1. 5

4. ANALYSIS OF THICK WALLED CYLINDER WITH INTERNAL CRACKS

Thick walled cylinders used in many industrial applications may have a chance of internal cracks. The orientation of cracks may differ based on the field of application of pressure vessels. So it is relevant to study about the stress intensity at the point of crack and predicts the failure of pressure vessels. This work aims at determining the variation of Stress Intensity Factors (SIF) at different crack lengths (crack may be given radially) by varying the internal pressures. By determining the SIF makes us to easily identify the stress concentrations in the cylinder and thereby predicting the strength of the cylinder. To analyse the cylinder with internal radial crack two dimensional modelling is done in ANSYS and results are obtained under varying internal pressures. The material selected is mild steel. The geometry is meshed using 8-node (PLANE 82) quadrilateral elements. The obtained results are graphically plotted to give better interpretation:

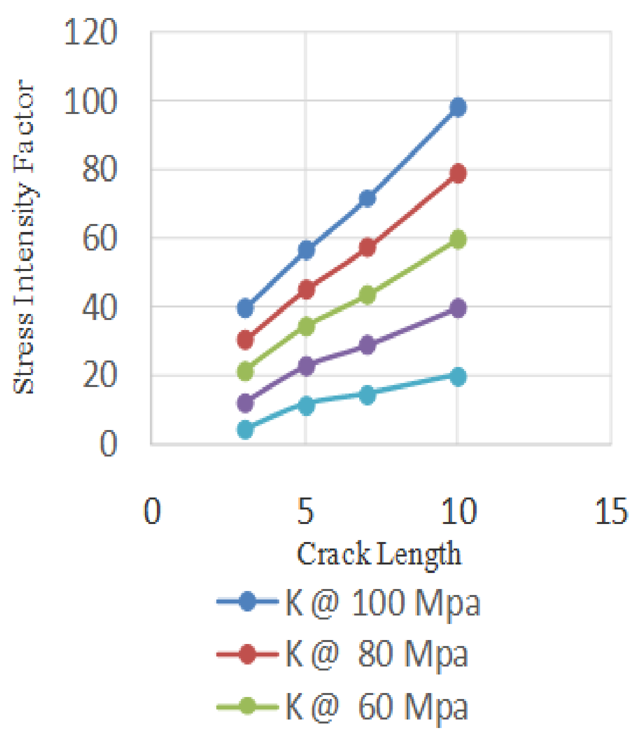


Fig. 7. Crack Length VS Stress Intensity Factor

Every material has a critical crack length beyond which the material fails suddenly. The critical crack length is determined theoretically in the current work obtained from the equation utilizing the fracture toughness of material. In the present work the critical crack length of Aluminium and Steel is determined at different internal pressures and the results are tabulated. The tabulated results are plotted down:

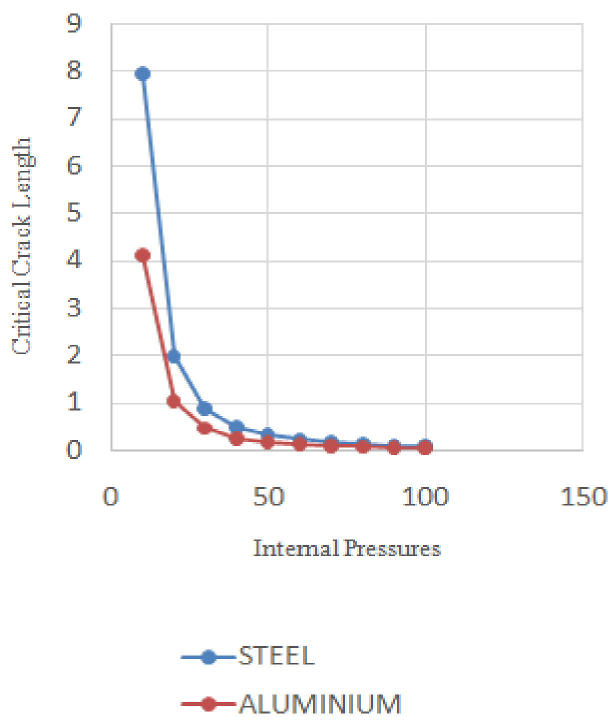


Fig. 8. Critical crack length for different materials

Cracks generated in thick cylinders may be of different orientations. They may be axially or radially present in the cylinder. A study was conducted in ANSYS on the crack orientation by comparing the Stress Intensity Factors of both cylinders with axial and radial cracks. The material selected is Steel.

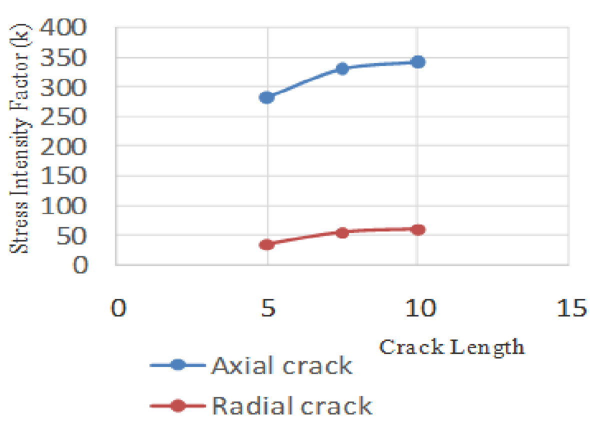


Fig. 9. Comparison of cracks based on SIF

For modelling the cylinder with radial crack two dimensional modelling was done and by applying sufficient internal pressures the SCF (k) was determined. For cylinder with axial cracks 3d modelling was implemented. This analysis on both cylinders with radial and axial cracks was useful in predicting the strength of the cylinder. The results were tabulated and graphically represented to give better interpretation. From the graphs, we see that the cylinder with axial cracks fails faster than radial cracks under same crack length.

5. METHODS TO IMPROVE BEARING CAPACITY OF PRESSURE VESSELS

Current industrial applications search pressure vessels with improved bearing capacity which opens up the chances of research in the area. The improved bearing capacity of the pressure vessels leads to better and efficient working of the pressure vessel under higher internal pressures. Two methods of improving bearing capacity is being discussed in this paper. They are:

5.1 Compound Cylinders

Compound cylinders are used in most industrial application because of its load bearing capacity. Basic difference between compound cylinder and single cylinder is that of their structure which means compound cylinder is a combination of two or more cylinders. The effect of contact pressure at the contact of two cylinders reduces the stress developed within the cylinders which increase its life.

We have analysed compound cylinder which is a combination of steel and PET (Polyethylene terephthalate) which is commonly used in industrial application of their properties like insulation, less effect to corrosion and high mechanical strength. Compound cylinder of STEEL and PET was modelled and analysed in FEA and the following characteristic curves were obtained. In compound cylinder the higher stress is developed in inner cylinder than the outer jacket and a sudden change can be observed. The stress developed in the inner cylinder will also vary with the thickness.

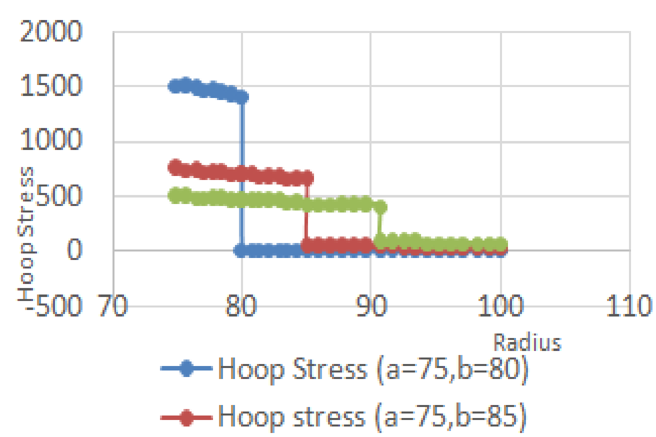


Fig. 10. Hoop stress variation of Steel-PET compound cylinder

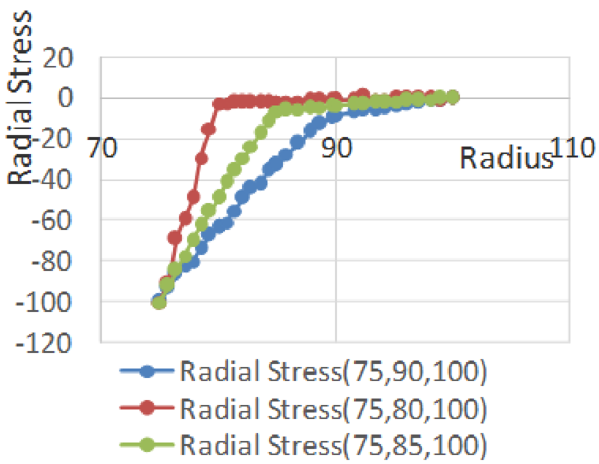


Fig. 11. Radial stress variation of Steel-PET compound cylinder

Both compound cylinder and single cylinder was subjected to axial crack of 1mm and the stress concentration factors were obtained and on comparing the values we could see that SCF value for compound cylinder is much less than that of the single cylinder. From these values we can predict compound cylinders with axial crack have longer life that single cylinders with axial crack of same length.

Table 1. Stress concentration factor of single and compound cylinders

	SCF(K)
Compound cylinder(steel &steel)	
a=75,c=80,b=100	k=7.94
a=75,c=85,b=100	k=13.56
a=75,c=90,b=100	k=20.64
Single cylinder(steel)	
a=75,b=100	k= 44.77

5.2. Autofrettage process

Autofrettage is the process of developing residual stresses within the walls of thick cylinders by sudden loading of large pressure (to cause yielding within the wall) and sudden releasing the pressure to develop compressive residual circumferential stress up to a certain radial depth defined as autofrettage radius. The residual stress developed reduces tensile stress developed due to the operating pressure. This process in turn improves the load bearing capacity of the cylinder.

Autofrettage radius can be obtained from the equation [8, 10]:

$$r_{a,opt} = r_i \times e^n \qquad n = \frac{P_{opr}}{\sigma_y}$$

Where,

Pressure needed to be applied for autofrettage:

$$P_{app} = \frac{\sigma_y}{2}\left[1 - \frac{e^{2n}}{k^2} + 2n\right] \quad \text{Or}$$

$$\frac{P_{app}}{P_{opr}} = \left[1 + \frac{k^2 - e^{2n}}{2nk^2}\right]$$

Stresses developed in the cylinder:
Due to operating pressure:

$$\sigma_r = \frac{P_i}{k^2-1}\left[1 - \frac{r_o^2}{r^2}\right]$$

$$\sigma_r = \frac{P_i}{k^2 - 1}\left[1 + \frac{r_o^2}{r^2}\right]$$

Residual stress developed due to autofrettage:

For plastic region: ie, $r_i \leq r \leq r_a$

$$\sigma_{rad,res} = \frac{\sigma_y}{2}\left\{\left[2\ln\left(\frac{r}{r_a}\right) - 1 + \frac{m^2}{k^2}\right] - \left[2\ln(m+1) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\left(1 - \frac{r_o^2}{r^2}\right)\right\}$$

For elastic region: ie $r_a \leq r \leq r_o$

$$\sigma_{hoop,res} = \frac{\sigma_y}{2}\left\{\left[2 + 2\ln\left(\frac{r}{r_a}\right) - 1 + \frac{m^2}{k^2}\right] - \left[2\ln(m+1) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\left(1 + \frac{r_o^2}{r^2}\right)\right\}$$

$$\sigma_{rad,res} = \frac{\sigma_y}{2}\left(1 - \frac{r_o^2}{r^2}\right)\left\{\left[\frac{m^2}{k^2}\right] - \left[1 + 2\ln(m) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\right\}$$

$$\sigma_{hoop,res} = \frac{\sigma_y}{2}\left(1 + \frac{r_o^2}{r^2}\right)\left\{\left[\frac{m^2}{k^2}\right] - \left[1 + 2\ln(m) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\right\}$$

By substituting $r = r_a$, the residual stress at the junction radius r_a is obtained:

$$\sigma_{rad,res} = \frac{\sigma_y}{2}\left(1 - \frac{k^2}{m^2}\right)\left\{\left[\frac{m^2}{k^2}\right] - \left[1 + 2\ln(m) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\right\}$$

$$\sigma_{hoop,res} = \frac{\sigma_y}{2}\left(1 + \frac{k^2}{m^2}\right)\left\{\left[\frac{m^2}{k^2}\right] - \left[1 + 2\ln(m) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2-1}\right)\right\}$$

Total stress developed:

$$\sigma_{rad,total} = \sigma_{rad} + \sigma_{rad,residual}$$

$$\sigma_{hoop,total} = \sigma_{hoop} + \sigma_{hoop,residual}$$

The notations are defined as:

r_i = internal radius
r_o = outer radius
σ_y = yield strength
P_{opr} = operating pressure
$K = radius\ ratio = \frac{r_o}{r_i}$

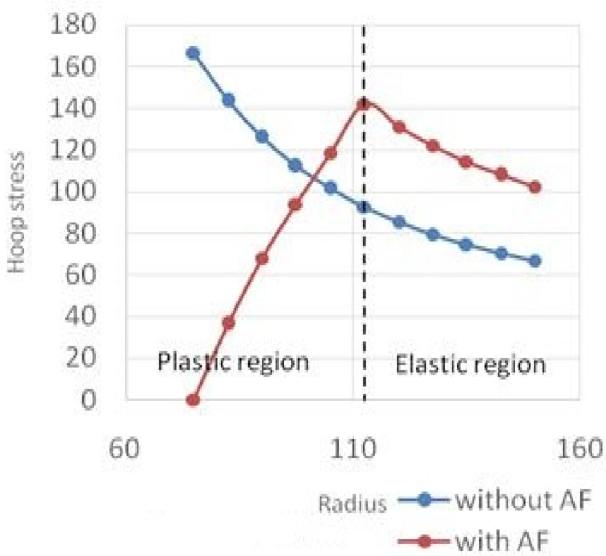


Fig. 12. Hoop stress vs Radius

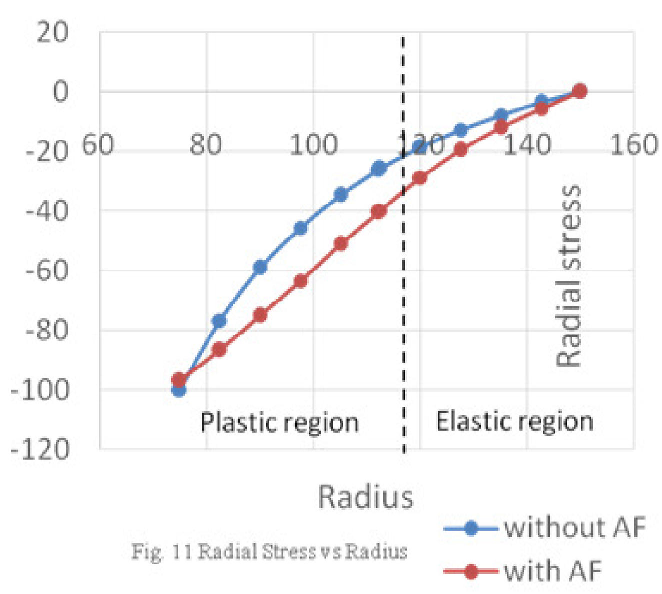


Fig. 13. Radial stress vs Radius

From the graphs figure 12 and figure 13 we can observe that hoop stress and radial stress are lesser than the actual values obtained without the process of autofrettage. The maximum hoop stress is lesser and is obtained not at internal radius but at autofrettage radius. As we move from r_i to r_o we can observe that the hoop stress increases up to autofrettage radius and reaches maximum value which then reduces. The radial stress distribution is also different since its values are lesser than the actual stress values without autofrettage.

6. CONCLUSIONS

Thick cylinders were subjected to high internal pressures under various crack orientations and Autofrettage analysis was also done and various conclusions were drawn from data's and graphs.

- There is noted increase in the Hoop stress with the decrease in radius ratio (wall thickness)
- Stress intensity factor (SCF) increases with increase in pressures at a particular crack length. It also increases with increase in crack length.
- By analytical equations it was found that critical crack length of Steel is more than that of Aluminium at particular pressure thus making steel preferable.
- For a thick cylinder with same internal crack length, the axial crack is more vulnerable to failure than radial crack because the SIF is more for axial crack.
- For same wall thickness compound cylinders are preferred than simple cylinders because of the reduction in SIF of compound cylinders.
- Autofrettage increase the pressure bearing capacity as it decreases the hoop and radial stress at the vicinity of the inner radius.

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