

Optimized Design of Machine Elements Subjected to Fatigue Loading Using Fuzzy Logic

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Abstract: A technique to perform design calculations on imprecise representations of parameters has been developed and is presented. The level of imprecision in the description of design elements is typically high in the preliminary phase of engineering design. This imprecision is represented using the fuzzy calculus. Calculations can be performed using this method, to produce (imprecise) performance parameters from imprecise (input) design parameters. In this work, application of fuzzy logic to the design of helical compression spring is explored.

1. INTRODUCTION

The methods and tools for designing mechanical components are many and varied. Most of the designers and authors of text-books on machine design [5], [6] adopt their own ways. The traditional methods based on deterministic approach have been replaced by probabilistic approach and now, with the invent of fuzzy logic [zadeh] as a powerful tool to handle uncertainty it is becoming more and more acceptable to the designers of engineering systems.

Present work is an attempt to substantiate the design process using fuzzy sets by applying the fuzzy extension principle to the design of mechanical components. As an example spring is designed for dynamic loading using the conventional method and by applying the fuzzy principle.

2. FUZZY THEORY FOR DESIGN OF COMPONENTS

Variables involved in design, referred to as parameters, are classified into input parameters, output parameters, output parameters and performance parameters. An input parameter is one whose value is determined during the design process. An output parameter is any parameters which is functionally dependant on the input parameters and possibly on some performance parameter, but are not subjected to any specified functional requirements. Once the relevant parameters in a particular design problem are determined, the first step in applying fuzzy theory to this problem is to express the preferences regarding the values of the input parameter by appropriate fuzzy sets.

3. DESIGN OF HELICAL COIL SPRING

THE SPRING MUST GIVE A MINIMUM FORCE OF 267 N AND A MAXIMUM FORCE OF 667 N OVER A DYNAMIC DEFLECTION OF 25.4MM. THE FORCING FREQUENCY IS 1000 RPM. A 10-YEAR LIFE OF 1-SHIFT OPERATION IS DESIRED. MUSIC WIRE (ASTM A228) WILL BE USED, SINCE THE LOADS ARE DYNAMIC.

3.1 Design of Spring by Conventional Design Method

1. The number of cycles that the spring will see over its design life is given by

$$N_{life} = 1000 \frac{rev}{min} \left(\frac{60 min}{hr} \right) \left(\frac{2080 hr}{shift - yr} \right) (10yr) \\ = 1.248 \times 10^9 cycles$$

This large a number requires that an endurance limit for infinite life be used.

2. The alternating and mean forces are calculated

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{667 - 267}{2} = 200 N$$

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{667 + 267}{2} = 467 N$$

3. A spring index of 9 is taken and calculate the mean coil diameter D

$$D = Cd = 9(5.5) = 49.5mm$$

4. The direct shear stress factor K_s is calculated

$$K_s = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{9} = 1.056$$

Stress at initial deflection is given by

$$\tau_i = K_s \frac{8F_i D}{\pi d^3} = 1.056 \frac{8(267)(49.5)}{\pi(5.5)^3} = 213.62 N/mm^2$$

And the mean stress is given by

$$\tau_m = K_s \frac{8F_m D}{\pi d^3} = 1.06 \frac{8(467)(49.5)}{\pi(5.5)^3} = 373.63 \text{ N/mm}^2$$

The Wahl factor K_w is calculated

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4(9) - 1}{4(9) - 4} + \frac{0.615}{9} = 1.162$$

And is used to calculate the alternating shear stress

$$\tau_a = K_w \frac{8F_a D}{\pi d^3} = 1.162 \frac{8(200)(49.5)}{\pi(5.5)^3} = 176.07 \text{ N/mm}^2$$

The ultimate tensile strength of this music-wire material is determined

$$\sigma_{ut} = Ad^b = 2153.5 (5.5)^{-0.1625} = 1632.44 \text{ N/mm}^2$$

The ultimate shear strength is given by

$$\sigma_{us} = 0.67 \sigma_{ut} = 0.67(1632.44) = 1093.73 \text{ N/mm}^2$$

And the torsional yield strength is given by

$$\sigma_{ys} = 0.60 (1632.44) = 979.46 \text{ N/mm}^2$$

The wire endurance limit for peened springs in repeated loading is

$$\sigma_{ew} = 465 \text{ N/mm}^2$$

And converting it to a fully reversed endurance strength

$$\begin{aligned} \sigma_{es} &= 0.5 \left(\frac{\sigma_{ew} \sigma_{us}}{\sigma_{us} - 0.5 \sigma_{ew}} \right) \\ &= 0.5 \left(\frac{465 \times 1093.73}{1093.73 - 0.5(465)} \right) = 295.27 \text{ N/mm}^2 \end{aligned}$$

The safety factor is calculated

$$\begin{aligned} N_{fs} &= \frac{\sigma_{es}(\sigma_{us} - \tau_i)}{\sigma_{es}(\tau_m - \tau_i) + \sigma_{us}\tau_a} \\ &= \frac{295.27(1093.73 - 213.62)}{295.27(373.63 - 213.62) + 1093.73(176.07)} = 1.08 \end{aligned}$$

This is obviously not an acceptable design. The spring index is decreased from 9 to 7, keeping all other parameters the same, an acceptable design is obtained in this case with $N_{fs} = 1.4$. A summary of the changed value is

$$C = 7, D = 38.5\text{mm}, K_w = 1.212, K_s = 1.07, \tau_i = 168.35 \text{ N/mm}^2, \tau_a = 142.84 \text{ N/mm}^2, \tau_m = 294.45 \text{ N/mm}^2, N_{fs} = 1.4$$

The spring rate is defined from the two specified forces at their relative deflection

$$k = \frac{\Delta F}{y} = \frac{667 - 267}{25.4} = 15.75 \text{ N/mm}$$

Number of active coils is given by

$$k = \frac{d^4 G}{8D^3 N_a} \text{ or}$$

$$N_a = \frac{d^4 G}{8D^3 k} = \frac{(5.5)^4 79290}{8(38.5)^3 15.75} = 10$$

The free length is found to be

$$\begin{aligned} L_f &= L_s + y_{clash} + y_{working} + y_{initial} \\ &= d(N_a + 2) + 0.15y + 25.4 + 267/k \\ &= 112.16\text{mm} \end{aligned}$$

As the spring is to be put in a hole, hence there will be no buckling.

3.2 Design by Application of Fuzzy Theory

The various input parameters in this design problem are F , D and d . Out of these three input parameters, the value of F is specified in the problem and hence, we need only to desire. Though the spring is to fit in a hole but if large clearances are used, there are chances of buckling and hence, the lower value of D i.e. the mean diameter is fixed. Also, the higher limit is fixed after allowing for the clearance in accordance with the minimum wire diameter chosen.

Thus, the wire diameter and the mean coil diameter are represented by the following fuzzy sets respectively as

$$\begin{aligned} d &= \{3, 4.5, 6.5\} \\ &\text{and} \\ D &= \{37, 41, 45\} \end{aligned}$$

where, this set represents the membership function of these parameters based on the available standard values, rather than the triangular fuzzy number.

The performance parameter in this problem can be taken to be the factor of safety N_{fs} whose dependence on the input variables d and D are given as

$$N_{fs} = \frac{155.75Ad^b}{0.67Ad^b - 232.5} \left(0.67Ad^b - \frac{K_s 8F_i D}{\pi d^3} \right) \frac{1}{\frac{155.75Ad^b}{0.67Ad^b - 232.5} \left(\frac{K_s 8F_m D}{\pi d^3} - \frac{K_s 8F_i D}{\pi d^3} \right) + 0.67Ad^b \frac{K_w 8F_a D}{\pi d^3}}$$

be ensured that the factor of safety is sufficient. A specification of the range of factor of safety that is sufficient is represented by a fuzzy set NE whose membership is specified in the Fig. 3 and represented as

$$NE = \begin{cases} 0 & f < 1.2 \\ \frac{f - 1.2}{1.4 - 1.2} & 1.2 < f < 1.4 \\ 1 & f > 1.4 \end{cases}$$

Now, the various α -cuts of the two parameters d and D are represented for the discrete values i.e. 0.2, 0.4, 0.6, 0.8, 1.0 etc. as

$$^{0.2}d = \{3, 6.5\} \quad ^{0.2}D = \{37, 45\}$$

$$^{0.4}d = \{3.5, 6\} \quad ^{0.4}D = \{38, 44\}$$

$$^{0.6}d = \{3.75, 5.5\} \quad ^{0.6}D = \{39, 43\}$$

$$^{0.8}d = \{4.0, 5.0\} \quad ^{0.8}D = \{40, 42\}$$

$$^{1.0}d = \{4.5\} \quad ^{1.0}D = \{41\}$$

Corresponding to these values of the input parameters the discrete α -cuts of the performance parameter N_{fs} , are given as

$$^{0.2}f = \{0.008, 2.494\} \quad ^{0.4}f = \{0.155, 1.894\}$$

$$^{0.6}f = \{0.254, 1.392\} \quad ^{0.8}f = \{0.368, 0.978\}$$

$$^{1.0}f = \{0.638\}$$

From the values of the discrete α -cuts of the induced fuzzy set G on the performance parameter, the membership function of the fuzzy set is represented in Fig.3.

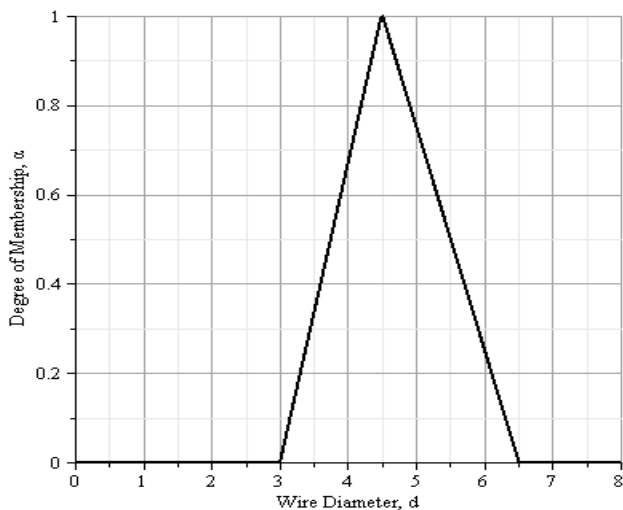


Fig. 1: Fuzzy sets for Wire Diameter, d

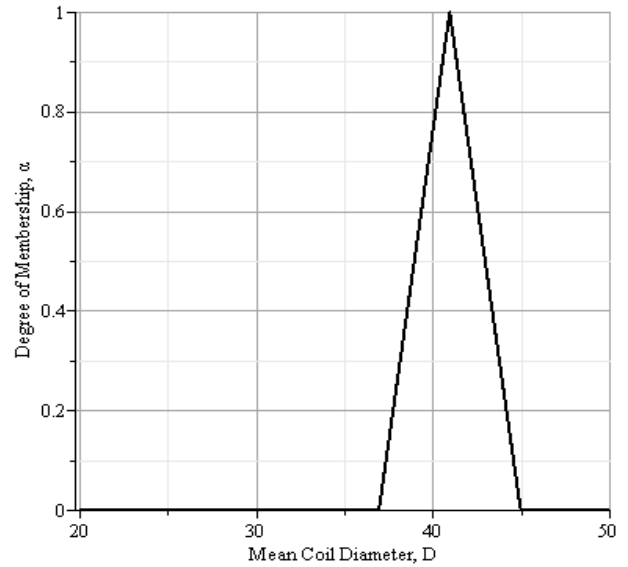


Fig. 2: Fuzzy sets for Mean Coil Diameter, D

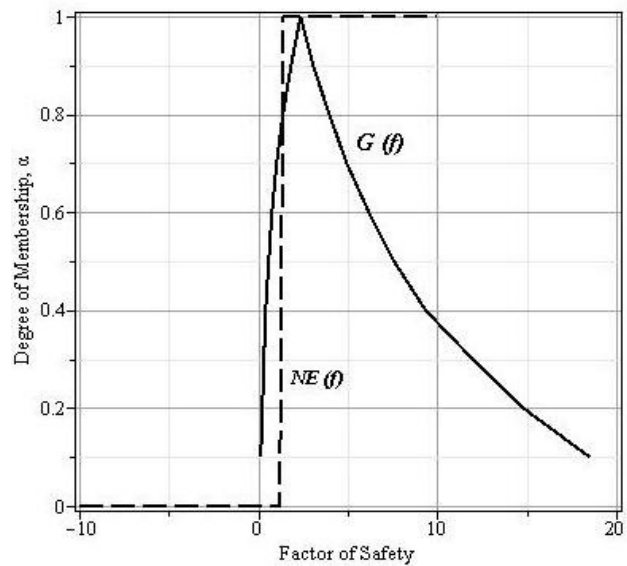


Fig. 3: Fuzzy Sets for Performance Parameter NE and Induced Membership Function G

Hence, the optimal values of the input parameters based on the fuzzy theory are

$$\text{Wire diameter, } d = 5.5 \text{ mm}$$

$$\text{Mean coil diameter, } D = 39 \text{ mm}$$

4. CONCLUSION

From the design example of spring, it can be seen that the choice of the input parameters can be made with a greater degree of confirmation in the fuzzy method than in the

conventional method. Also, the number of iterations to be made for reaching at the optimal design is lesser than that for the conventional process. The method based on fuzzy logic has proven to be effective and scope exists for its application to other mechanical components in the similar manner.

REFERENCES

- [1] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, New Delhi: Prentice Hall of India Private Limited, 2000.
- [2] K. L. Wood and E. K. Antonsson, "Computations with Imprecise Parameters in Engineering Design: Background and Theory," *ASME Journal of Mechanisms, Transmission and Automation in Design*, vol. 111, no. 4, pp. 616-625, 1989.
- [3] K. N. Otto and E. K. Antonsson, "Extensions to the Taguchi Method of Product Design," *ASME Journal of Mechanical Design*, vol. 115, no. 1, pp. 5-13, 1993a.
- [4] K. N. Otto and E. K. Antonsson, "Turning Parameters in Engineering Design," *ASME Journal of Mechanical Design*, pp. 14-19, 1993b.
- [5] V. B. Bhandari, *Design of Machine Elements*, New Delhi: Tata McGraw-Hill, 2007.
- [6] J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design*, New Delhi: Tata McGraw-Hill, 2003.
- [7] P. C. o. Technology, *Design Data Book*, Coimbtore: DPV Printers, 2002.
- [8] Norton, R. L. (2000). *Machine Design an integrated approach* (Second Edition). New Delhi: PEARSON