

A Noble Hybrid Concept for MIMO System Modeling using Bacterial Foraging Optimization Technique

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Abstract: A new concept for system reduction of large control system is presented here. In this paper, basic approach for model reduction is proposed using combined advantages of stability equation method and error minimization by Bacterial Foraging optimization (BFO). In this algorithm, coefficients of denominator polynomial of reduced system are determined by stability equation method and the numerator coefficients are calculated by minimizing the integral square error (ISE) between the step response of original high order system and reduced model using BFO technique. The proposed concept in this paper is illustrated through numerical example and a comparative study has been made with some others existing concepts. The method preserves the stability in the reduced model of original system. It is also observed that my proposed concept gives low ISE value than existing algorithms. The concept is simple and computer oriented. The proposed concept has been implemented in Mat-lab 2010 environment on a Pentium-IV computer.

Keywords: System reduction, Stability, Bacterial Foraging optimization, Integral square error.

1. INTRODUCTION

The system modeling of any physical system is considered to be an important and challenging issues in many diverse field of Science, engineering and technology.. A model is often very complicated to be used in real life situations. So, approximation steps based on physical assumptions or mathematical approaches are adopted to achieve simpler models than the original one. The system approximation theory is very useful to technocrats and scientists working in the process industries. In control engineering field, system reduction techniques are fundamental for the design of controllers where numerically complicated procedures are involved.

A large number of text books [1-5], bibliography [6-9] and research papers [10-20] have been published in the field of system reduction of physical systems over last couples of decades.

2. SYSTEM APPROXIMATION ALGORITHM

The method consists of two steps for reducing the order of high order original system, as stated below:

STEP-1: Determination of the denominator coefficients of reduced model using Stability Equation Method.

For stable high order system $G(s)$, the denominator $D(s)$ of the high order system is decomposed in the even and odd components in the form of stability equations as:

$$D_{\text{even}}(s) = \sum_{i=0,2,4}^n d_i s^i = d_0 \prod_{i=1}^{m_1} \left(1 + \frac{s^2}{z_i^2}\right) \quad (1)$$

$$D_{\text{odd}}(s) = \sum_{i=1,3,5}^n d_i s^i = d_0 \prod_{i=1}^{m_2} \left(1 + \frac{s^2}{p_i^2}\right) \quad (2)$$

where m_1 and m_2 are integer parts of $n/2$ and $(n-1)/2$, respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2 \dots$

Now, by neglecting the factors of larger magnitudes of z_i^2 and p_i^2 in equation (1) and (2), the stability equations for r^{th} order system are obtained as below:

$$D_{\text{even}}^r(s) = d_0 \prod_{i=1}^{m_3} \left(1 + \frac{s^2}{z_i^2}\right) \quad (3)$$

$$D_{\text{odd}}^r(s) = d_1 s \prod_{i=1}^{m_4} \left(1 + \frac{s^2}{p_i^2}\right) \quad (4)$$

where, m_3 and m_4 are the integer parts of $r/2$ and $(r-1)/2$ respectively.

After adding two reduced order model stability equations given in equation (3) and (4) and then proper normalizing it, the r^{th} order denominator $D_r(s)$ of reduced model can be obtained as:

$$D_r(s) = D_{\text{even}}^r(s) + D_{\text{odd}}^r(s) = \sum_{i=0}^{r-1} d_i s^i + s^r \quad (5)$$

Thus, the denominator polynomial is now known as:

$$D_r(s) = f_0 + f_1 s + f_2 s^2 + \dots + f_{n-1} s^{r-1} + s^r \quad (6)$$

STEP-2: Determination of the numerator coefficients of the reduced order model R(S) By Bacterial Foraging (BFO) Method.

The Bacterial Foraging optimization (BFO) technique is used to minimize the objective function (J) which is the integral square error (ISE) between the transient responses of original high order model $G(s)$ and reduced order model $R(s)$. The ISE is defined by:

$$\text{ISE (J)} = \int_0^{\infty} [y(t) - y_r(t)]^2 dt \quad (7)$$

where $y(t)$ and $y_r(t)$ are the unit step responses of original high order system $G(s)$ and reduced order system $R(s)$, respectively. The parameters to be determined are the numerator polynomial coefficients e_i ($i = 0, 1, \dots, (r-1)$)

3. SYSTEM UNDER STUDY

Consider a sixth-order two- input two- output system from literature [3] described in the form of transfer function matrix as:

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (8)$$

or,

$$[G(s)] = \frac{1}{D(s)} \begin{bmatrix} c_{11}(s) & c_{12}(s) \\ c_{21}(s) & c_{22}(s) \end{bmatrix} \\ = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

where,

$$G_{11}(s) = c_{11}(s)/D(s), \quad G_{12}(s) = c_{12}(s)/D(s), \\ G_{21}(s) = c_{21}(s)/D(s), \quad G_{22}(s) = c_{22}(s)/D(s). \quad (9)$$

Here,

$$D(s) = 6000 + 13100s + 10060s^2 \\ + 3491s^3 + 571s^4 \\ + 41s^5 + s^6 \quad (10)$$

$$c_{11}(s) = 6000 + 7700s + 3610s^2 \\ + 762s^3 + 70s^4 \\ + 2s^5$$

$$c_{12}(s) = 2400 + 4160s + 2182s^2 \\ + 459s^3 + 38s^4 \\ + s^5$$

$$c_{21}(s) = 3000 + 3700s + 1650s^2 \\ + 331s^3 + 30s^4 \\ + s^5$$

$$c_{22}(s) = 6000 + 9100s + 3660s^2 \\ + 601s^3 + 42s^4 \\ + s^5$$

(11)

The proposed method using Bacterial Foraging (BFO) technique is applied to each element i.e. [$G_{11}(s), G_{12}(s), G_{21}(s)$ and $G_{22}(s)$] of the transfer function matrix of two-input and two- output system (multi-variable) system in Eq. (8) to obtain its reduced order model as below:

$$[R(s)] = \frac{1}{\tilde{D}(s)} \begin{bmatrix} e_{11}(s) & e_{12}(s) \\ e_{21}(s) & e_{22}(s) \end{bmatrix} = \begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix}$$

where,

$\tilde{D}(s) = s^2 + 1.34952s + 0.6181$ is the common denominator polynomial obtained by stability equation method and numerator polynomials [$e_{11}(s), e_{12}(s), e_{21}(s)$ and $e_{22}(s)$] of the reduced model in equation (12) are obtained using BFO as below:

$$e_{11}(s) = 0.790519s + 0.659123$$

$$e_{12}(s) = 0.4819501s + 0.271751$$

$$e_{21}(s) = 0.3918093s + 0.324120$$

$$e_{22}(s) = 1.10775211s + 0.638205$$

The step responses of original and reduced order model are obtained and depicted in Fig. 1(a) to 1(d). The parameters of BFO are suitably chosen to obtain the optimum or global value. In this work, the parameters values are $S = 8, p = 2, N_c = 8, N_s = 3, N_{re} = 6, N_{ed} = 3$ and $p_{ed} = 0.25$.

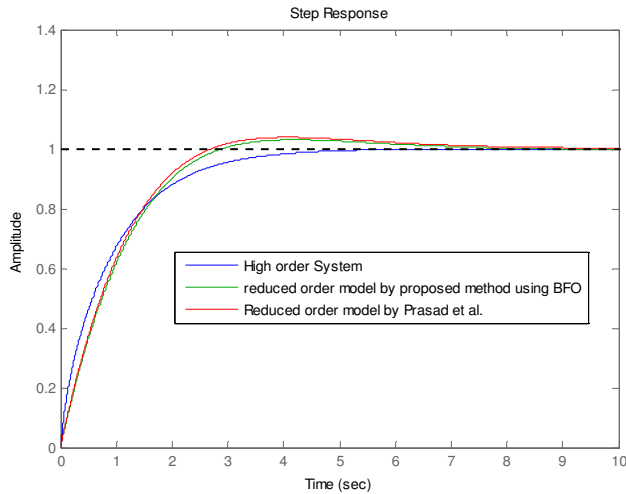


Fig.1 (a): Comparison of step responses of reduced $[R_{11}(s)]$ and HOS $[G_{11}(s)]$

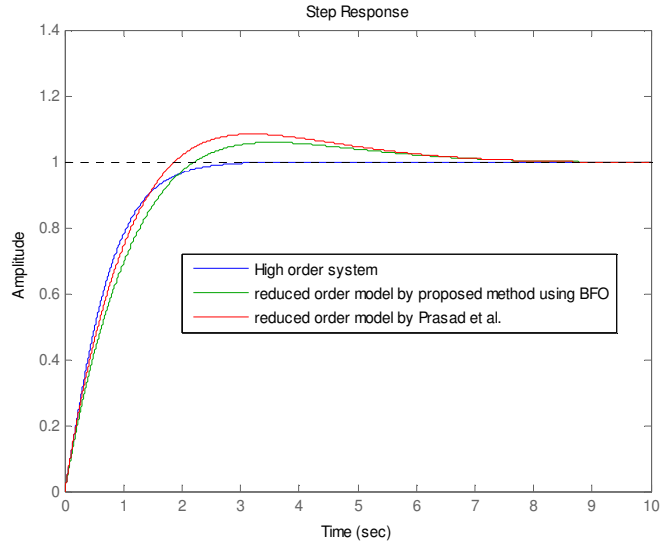


Fig.2.6 (d): Comparison of step responses of reduced $[R_{22}(s)]$ and HOS $[G_{22}(s)]$

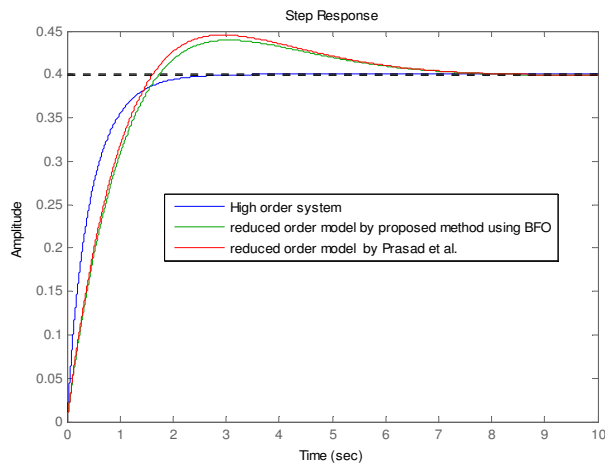


Fig.2.6 (b): Comparison of step responses of reduced $[R_{12}(s)]$ and HOS $[G_{12}(s)]$

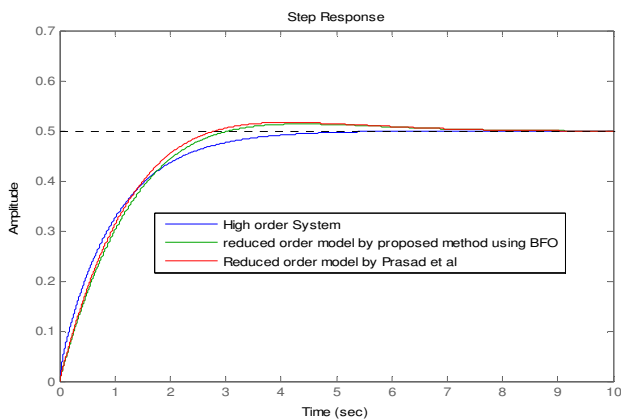


Fig.2.6 (c): Comparison of step responses of reduced $[R_{21}(s)]$ and HOS $[G_{21}(s)]$

A comparative study of the proposed method with some well known existing model reduction techniques for 2nd order reduced order model of two inputs two output system is made in Table 1. An error performance index integral square error (ISE) between the transient parts of original and reduced order system is determined to measure the goodness of the reduced order model and is calculated by

$$ISE (J) = \int_0^{\infty} [g_{ij}(t) - r_{ij}(t)]^2 dt \tag{13}$$

where $g_{ij}(t)$ and $r_{ij}(t)$ are the unit step responses of original and reduced order models respectively.

Table 1: Comparison of reduced order model methods

Model Reduction Methods	ISE for all $r_{ij} (i = 1, 2 ; j = 1, 2)$			
	r_{11}	r_{12}	r_{21}	r_{22}
Proposed Algorithm	0.011517	0.007521	0.002106	0.017903
Prasad and Pal	0.136484	0.002446	0.040291	0.067902
Safonev et.al.	0.590617	0.037129	0.007328	1.066123
Prasad et al.	0.030689	0.000256	0.261963	0.021683

It is also observed from Table-1 that the proposed method gives low value of J for all $r_{ij} (i=1, 2; j=1, 2)$ in comparison to the other well known existing order reduction techniques.

4. CONCLUSION

A noble approach for system modeling of high order LTIC control system by hybrid method using bio-inspired Bacterial Foraging optimization (BFO) has been proposed. In this algorithm, coefficients of denominator polynomial of reduced order transfer function model are obtained by stability equation method and the numerator coefficients are determined by Bacterial Foraging Optimization (BFO) technique. The proposed method is discussed in this paper is illustrated through numerical example and compared with some well known existing techniques.. These methods preserve steady state value and stability in the reduced models of original systems. It is also observed that my proposed method give low value of ISE than existing methods. The proposed algorithms have been implemented in Mat-lab 2010 environment on a Pentium-IV computer.

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