

# Study of Free Vibration Analysis of Laminated Composite Plates with Skew Cut-outs based on FSDT

Harsh Kumar Bhardwaj<sup>1</sup>, Jyoti Vimal<sup>2</sup>, Avadesh Kumar Sharma<sup>3</sup>

<sup>1,2,3</sup>Department of Mechanical Engineering,  
Madhav Institute of Technology and Science, Gwalior, India

**Abstract:** This paper presents the free vibration analysis of a composite laminated plate with skew cut-out, using finite element method (FEM), based on first order shear deformation theory (FSDT). A number of examples concerning different aspect ratios, different thickness ratios, different values of material properties, different size of cutouts, different number of layers and different boundary conditions, for a cross-ply composite laminate with skew hole are considered. Convergence study has been carried out by comparing the results obtained to the numerical results of previous work available in the literature. Non-dimensional frequencies decrease with increasing the size of the plate and thickness ratio of the plate. Non-dimensional frequencies increase with increasing the cut-out size, the number of laminates of the plate and the modulus ratio of the plate.

**Keywords:** Free Vibration, FEM, Composite Laminated plate, Cut-out.

## 1. INTRODUCTION

Composite materials are formed by combining two or more materials together to form an overall structure that is better in physical or chemical properties than the sum of the individual components. Composite materials are generally used for building, bridges and many engineering fields such as aerospace, aviation, chemical and nuclear engineering. Many studies have been devoted for free vibration analysis of laminated composite plates with cutouts. Aksu and Ali [1] described a method for the prediction of dynamic characteristics of rectangular plates with cutouts. Lee and Lim [2] studied the free vibration of isotropic and orthotropic square plates with square cutouts subjected to in-plane forces. Pandit et al. [3] studied the free vibration analysis of laminated composite rectangular plate using FEM. Sivakumar et al. [4] investigate the free vibration analysis of composite plates in the presence of cut-outs undergoing large amplitude oscillations. The Ritz finite element model using a nine node C O continuity, isoperimetric quadrilateral element along with a higher order displacement theory which accounts for parabolic variation of transverse shear stresses is used to predict the dynamic behavior. Lee [5] studied the finite element dynamic stability analysis of laminated composite skew structures subjected to in-plane pulsating forces. Lahouel

and Guenfoud [6] studied the vibration analysis of symmetric angle-ply laminated composite plates with and without square hole when subjected to compressive loads. Brethee [7] studied the effects of various plate parameters on free vibration analysis of symmetric and anti-symmetric laminated composite plates with a cut-out at the centre.

## 2. MATERIALS AND METHOD

The finite element software (ANSYS) is used with the aim of analyzing. In addition SHELL 281 is suitable for analyzing thin to moderately-thick shell structures. The element has eight nodes with six degree of freedom at each node. SHELL 281 may be used for layered applications for modeling composite shells or sandwich construction.

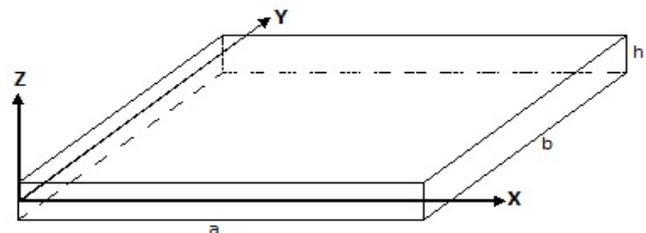


Figure 1 Geometry of the composite laminated composite plate

Considering the FSDT given by Reddy [8], the displacement fields at time  $t$  are expressed as follows:

$$u(x, y, z, t) = u_0(x, y, t) + z\phi_x(x, y, t), \quad (1)$$

$$v(x, y, z, t) = v_0(x, y, t) + z\phi_y(x, y, t), \quad (2)$$

$$w(x, y, z, t) = w_0(x, y, t). \quad (3)$$

## 3. NUMERICAL RESULTS AND DISCUSSION

Different cases of cross-ply composite laminates with skew type of cut-outs are examined here. Without loss of generality equal no. of grid points in  $x$ - and  $y$ - direction is assumed. To

define the boundary conditions along the edges the alphabet symbolism will be used, so that S-C-S-C indicates a plate with edge x=0 simply supported, edge y=0 clamped, edge x=a simply supported and edge y=b clamped.

**3.1 Convergence study**

To show the computational efficiency of FEM, a thin square plate composed with three orthotropic layers (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) is considered. The material properties of each layer are

$$\frac{E_{11}}{E_{22}} = 25, \quad G_{12} = G_{13} = 0.5E_{22}, \quad G_{23} = 0.2E_{22},$$

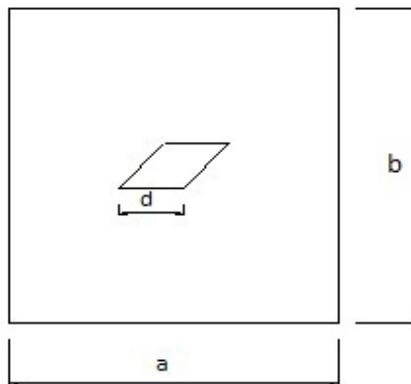
$$\nu_{12} = 0.25, \rho = 2700 \text{ kg/m}^3$$

The accuracy and convergence behaviors of the solutions for a square laminate plate with two orthotropic layers (0<sup>0</sup>/90<sup>0</sup>) with cut-out at the centre were investigated, the results of which are shown in Table 1. To compare the solutions the results of Pandit et al. [3] and Brethee [7] are also cited.

**Table 1 Convergence study of non-dimensional frequencies ( $\bar{\omega} = \frac{\omega a^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) for a cross-ply laminate (0<sup>0</sup>/90<sup>0</sup>) having a square cutout at the centre, ( $K_s=5/6, b/a=1, h/a=0.01$ ) for boundary condition i.e. CCCC with respect to the results given by [3] and [7]**

	1	2	Mode	4	5	6
Cut-out size			3			
0.2a*0.2a	9.827	25.833	26.277	40.301	54.818	63.052
[3]	9.11	25.63	25.80	38.11	54.23	60.64
[7]	9.071	25.057	25.057	37.643	53.154	59.856
0.4a*0.4a	10.050	21.382	21.984	37.752	46.409	65.219
[3]	9.12	20.25	20.34	35.67	44.76	61.81
[7]	9.061	19.930	19.930	35.070	42.866	60.270
0.6a*0.6a	12.402	20.022	20.525	34.749	37.438	56.576
[3]	11.31	18.69	18.71	32.81	34.34	53.11
[7]	11.085	18.173	18.173	31.560	33.920	51.712
0.4a*0.2a	9.63	22.35	25.74	39.38	53.40	64.98
[3]	8.85	21.31	27.82	39.12	51.22	62.14
[7]	8.76	20.57	54.35	36.57	50.83	60.81
0.8a*0.4a	10.83	13.21	30.58	34.90	55.77	66.97
[3]	9.72	11.81	27.36	31.15	50.63	60.37
[7]	9.60	11.54	27.03	30.63	49.42	59.27
0.6a*0.2a	9.57	17.18	25.90	36.97	52.70	62.58
[3]	8.54	15.87	25.45	34.87	51.26	60.13
[7]	8.49	15.27	25.08	34.02	50.24	59.21

**3.2 Vibration of composite laminated plate with skew cutout**



**Figure 2 Square plate with skew cutout**

Table 2 shows the variation of first ten non-dimensional frequencies with size ratio (d/b=0.2, 0.3, 0.4) of skew cut-out for a cross-ply square laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) of aspect ratio a/b=1 and thickness ratio h/b=0.01 for fully clamped boundary condition. The frequency for first two modes increases as the size ratio of cut-outs increase. Table 3 shows the variation of first ten non-dimensional frequencies with different number of layers for a thin cross-ply square laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup> ... ..) of aspect ratio a/b=1 and thickness ratio h/b=0.01, having skew cut-out of size ratio d/b=0.2 at the centre for fully clamped boundary condition. The frequency in all ten modes increases as the number of layers increases. Table 4 shows the variation of first ten non-dimensional frequencies with different boundary conditions (SSSS, SCSC and CCCC) for a thin cross-ply square laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) of aspect ratio a/b=1 and thickness ratio h/b=0.01, having skew cut-out of size ratio d/b=0.2 at the centre. The higher constraints at the edges results in higher frequencies.

**Table 2 Variation of first ten natural frequency parameters ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) with different values of size of hole (d/b=0.2, 0.3, 0.4) for a cross-ply laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) having skew cut-out at the centre, (K<sub>s</sub>=5/6, a/b=1, h/b=0.01, θ = 45<sup>0</sup>) for fully clamped boundary condition**

	Mode									
	1	2	3	4	5	6	7	8	9	10
d/b										
0.2	27.091	41.947	51.521	77.153	88.195	98.873	105.646	120.677	131.569	136.349
0.3	31.185	42.322	46.943	70.613	79.990	105.563	113.946	123.465	134.924	135.957
0.4	42.416	44.597	54.182	66.976	69.401	94.797	101.554	118.489	138.331	146.942

**Table 3 Variation of first ten natural frequency parameters ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) with different no. of layers (n=3, 5, 7, 9) for a cross-ply laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>.....) having skew cut-out at the centre, (K<sub>s</sub>=5/6, a/b=1, d/b=0.2, h/b=0.01, θ = 45<sup>0</sup>) for fully clamped boundary condition**

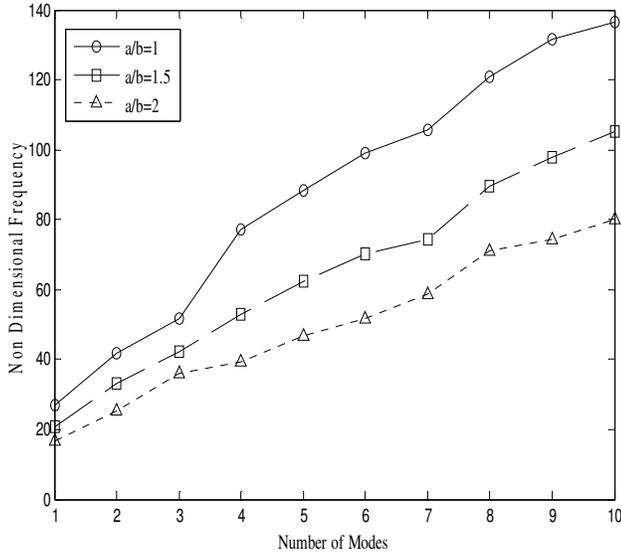
	Mode									
	1	2	3	4	5	6	7	8	9	10
No. of layers										
3	27.091	41.947	51.521	77.153	88.195	98.873	105.646	120.677	131.569	136.349
5	31.717	52.235	59.890	90.011	105.67	113.43	117.343	135.616	148.645	159.878
7	32.590	54.732	61.608	92.416	109.30	117.17	120.233	139.126	152.062	166.413
9	32.775	55.583	62.006	92.934	110.24	118.29	121.235	140.097	152.888	168.828

**Table 4 Variation of first ten natural frequency parameters ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) with different boundary conditions (SSSS, SCSC, CCCC) for a cross-ply laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) having skew cut-out at the centre, (K<sub>s</sub>=5/6, a/b=1, d/b=0.2, h/b=0.01, θ = 45<sup>0</sup>)**

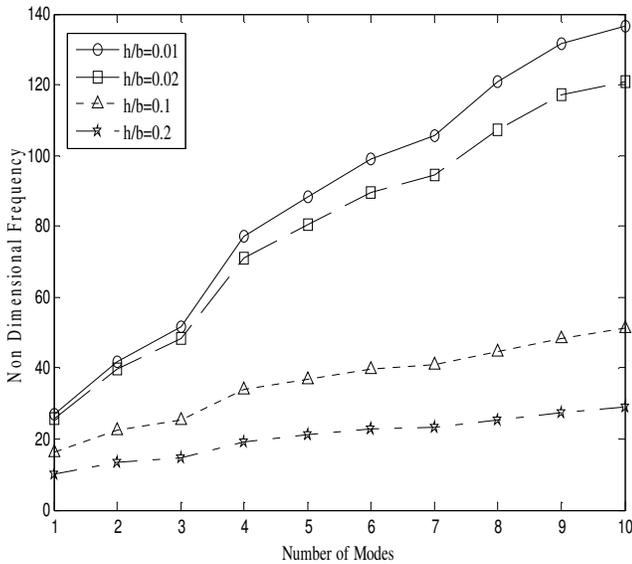
	Mode									
	1	2	3	4	5	6	7	8	9	10
B. C.										
SSSS	9.514	26.778	28.959	47.276	58.480	62.858	67.359	76.873	92.577	100.903
SCSC	19.481	32.568	42.354	63.049	64.421	77.202	94.990	102.794	108.815	127.935
CCCC	27.091	41.947	51.521	77.153	88.195	98.873	105.646	120.677	131.569	136.349

Figure 3 shows the variation of first ten non-dimensional frequencies with aspect ratio (a/b=1, 1.5, 2) for a cross-ply square laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) of thickness ratio h/b=0.01, having skew cut-out of size ratio d/b=0.2 at the centre for fully clamped boundary condition. The frequency in all modes decreases as the size of plate increases. Figure 4 shows the variation of first ten non-dimensional frequencies with thickness ratio (h/b=0.01, 0.02, 0.1, 0.2) for a cross-ply square laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) of aspect ratio a/b=1, having skew cut-out of size ratio d/b=0.2 at the centre for fully clamped

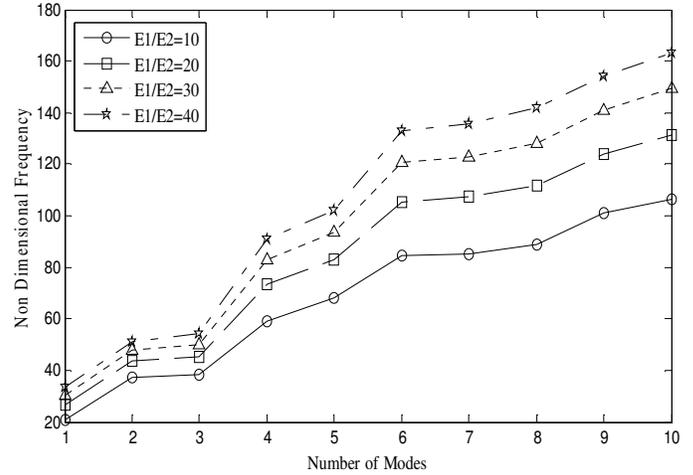
boundary condition. The frequency in all ten modes decreases as the thickness ratio increases. Figure 5 shows the variation of first ten non-dimensional frequencies with different material properties ( $\frac{E_{11}}{E_{22}} = 10, 20, 30, 40$ ) for a thin cross-ply square laminate (0<sup>0</sup>/90<sup>0</sup>/0<sup>0</sup>) of aspect ratio a/b=1 and thickness ratio h/b=0.01, having skew cut-out of size ratio d/b=0.2 at the centre for fully clamped boundary condition. The frequency in all ten modes increases as the modulus ratio increases.



**Figure 3** Variation of first ten natural frequency parameters ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) with size ratio ( $a/b=1, 1.5, 2$ ) for a cross-ply laminate ( $0^0/90^0/0^0$ ) having skew cut-out at the centre, ( $K_s=5/6, d/b=0.2, h/b=0.01, \theta = 45^0$ ) for fully clamped boundary condition



**Figure 4** Variation of first ten natural frequency parameters ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) with thickness ratio ( $h/b=0.01, 0.02, 0.1, 0.2$ ) for a cross-ply laminate ( $0^0/90^0/0^0$ ) having skew cut-out at the centre, ( $K_s=5/6, a/b=1, d/b=0.2, \theta = 45^0$ ) for fully clamped boundary condition



**Figure 5** Variation of first ten natural frequency parameters ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) with different material properties ( $\frac{E_{11}}{E_{22}} = 10, 20, 30, 40$ ) for a cross-ply laminate ( $0^0/90^0/0^0$ ) having skew cut-out at the centre, ( $K_s=5/6, a/b=1, d/b=0.2, h/b=0.01, \theta = 45^0$ ) for fully clamped boundary condition

**4. CONCLUSIONS**

The present study leads to the following conclusions for a composite laminated plate with skew cutouts:

1. Non-dimensional frequencies decrease with increasing the size of the plate and thickness ratio of the plate.
2. Non-dimensional frequencies increase with increasing the cut-out size, number of laminates of the plate and modulus ratio of the plate.
3. The boundary conditions of the plate play an important role in the free vibrations of the plate with cutouts. The Non-dimensional frequencies are higher for fully clamped boundary condition in comparison to other boundary conditions.

**REFERENCES**

[1] Aksu G. and Ali R. Determination of dynamic characteristics of rectangular plates with cutouts using a finite difference formulation. *J. Sound and Vibration* 1976, 44 (1), 147-158.

[2] Lee H. P. and Lim S.P. Free vibration of isotropic and orthotropic square plates with square cutouts subjected to in-plane forces. *Journal of Computers and Structures* 1992, 43 (3), 431-437.

[3] Pandit M. K., Haldar S. and Mukhopadhyay M. Free vibration analysis of laminated composite rectangular plate using finite element method. *Journal of Reinforced Plastics and Composites* 2007, 26 (1), 69-80.

- 
- [4] Sivakumar K., Iyengar K. and Deb K. Free vibration of laminated composite plates with cutout. *Journal of Sound and Vibration* 1999, 221 (3), 443-470.
- [5] Lee Sang-Youl. Finite element dynamic stability analysis of laminated composite skew plates containing cutouts based on HSDT. *Journal of Composites Science and Technology* 2010, 70 (8).
- [6] Lahouel Bahi-Eddine and Guenfoud Mohamed. Comparative analysis of vibration between laminated composite plates with and without holes under compressive loads 2013. World Academy of Science, Engineering and Technology.
- [7] Brethee F. Khaldoun. Free vibration analysis of symmetric and anti-symmetric laminated composite plates with a cutout at the centre. *Al-Qadisiya Journal for Engineering Sciences* 2009, 2.
- [8] Reddy J. N. *Mechanics of laminated composite plates and shells, Theory and Analysis (Second Edition)* London, CRC Press 2004.