Journal of Civil Engineering and Environmental Technology
Print ISSN: 2349-8404; Online ISSN: 2349-879X; Volume 1, Number 3; August, 2014 pp. 109-113
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http://www.krishisanskriti.org/jceet.html

Application of Curve Fitting in Indian Structures

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Abstract: **"Curve Fitting" is the process of constructing a curve or mathematical function that has the best fit to a series of data points, possibly subject to constraints. Curves such as parabola and hyperbola are used in architecture to design arches in buildings. They are known to be theoretically the strongest form of arches and commonly used in architectural design. Curves are preferred primarily as an aesthetic choice and at times make a building into something beautiful in a way rectilinear forms cannot. In this paper, we apply Campbell and Meyer's method of curve fitting to certain structures pertaining to Neo-Gothic Architecture and in Thermal Power Plant.**

Keywords: **Curve Fitting, Neo-Gothic Architecture, Conics, Least Squares Method**

1. INTRODUCTION

In architecture, curves are preferred mainly on the basis of distinguishing element of architecture. Curves such as parabolas and hyperbola are referred as conics. They are used in architecture to design arches in buildings and cooling towers in power plants. Oshin Vartanian, Psychologist of the University of Toronto compiled 200 images of interior architecture and explains about curves in architecture. He explains that curved design in architectural structures uses our brains to tug at our hearts. Structure flushed with curved design are more beautiful as it absorbs the brain activity and affects our feelings, which in return could drive our preference [6]. By establishing shapes in curves, we achieve arches.

An arch is a shape that resembles an upside down "U". Arches are used in architecture and civil engineering as a curved member to span an opening and to support loads. They are a passageway under a curved masonry construction [4, 8]. Arches have many forms but all fall in these basic categories - circular, pointed and parabolic. Arches can also be configured to produce vault and arcades. Arches with circular form also referred as rounded arch were commonly employed by the builders of ancient heavy masonry arches. Pointed arches were most often used by builders of Gothic-style architecture. The parabolic arch employs the principle that when weight is uniformly applied to an arch, the internal compression resulting from that weight will follow a parabolic profile.

Dirk Huylebrouck has studied about the shape of the arches constructed by Antoni Gaudi [2]. It would be interesting to find and discuss the shape of the curves of some of the architectural structures of India. In this paper, we apply the curve fitting method of Campbell and Meyer [1] to the Indian structures like University of Mumbai Library (Mumbai), Santhome Basilica (Chennai) and Sipat Thermal Power Plant (Chhattisgarh. "Curve Fitting" is the process of constructing a curve, or mathematical function that has the best fit to a series of data points, possibly subject to constraints. Hence we consider the data points on the curves found in these structures, fit a curve, preferably a conic and examine the goodness of fit.

2. CAMPBELL AND MEYER'S METHOD OF CURVE FITTING

Campbell and Meyer's theory on generalised inverse of a matrix has also been used in curve fitting [1, 2]. For example, if we are given some data points, we can find the conic section that provides the 'closet fit'. A variation which occurs quite frequently is that of trying to find the n[th] degree polynomial.

$$y = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2 + \cdots \widehat{\beta_n} x^n \qquad (1)$$

which best fits m data points (x_1, y_1), (x_2, y_2), … (x_m, y_m), $m > n + 1$.

We proceed by setting,

$$e_i = \sum_{j=0}^{n} \beta_j x_i^j - y_j, \quad \text{for each i} = 1, 2, \ldots \text{m} \qquad (2)$$

Thus,

$$\begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_m \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^n \\ & & & & \\ 1 & x_m & x_m^2 & \ldots & x_m^n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \beta_m \end{bmatrix} - \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_m \end{bmatrix}$$

or $\mathbf{e} = \mathbf{Xb} - \mathbf{y}$. If the restriction that $\|\mathbf{e}\|$ be minimal is imposed, then the closest nth degree polynomial to our data points has as its coefficients $\widehat{\beta}_0, \hat{\beta}_1, \dots \hat{\beta}_n$, where the $\hat{\beta}_i$'s are the components of a least square solution $\hat{\mathbf{b}} = \mathbf{X}^{+}\mathbf{y}$ of **Xb=y.**

To measure goodness of fit, we make use of Theorem 2.4.2 of [1] which states that the fraction

$\mathbf{R^2} = \|\mathbf{Xb}\|^2/\|\mathbf{y}\|^2$ gives us an accuracy of the proposed approximation. The notation X^+ represents the so called "generalized inverse" in the sense of Moore-Penrose, which corresponds to the regular inverse if it exists (that is, when X is invertible, and thus, when an exact solution can be computed).

We shall now apply this method for studying the shapes of the arches in the following structures:

3. APPLICATION 1: CURVE FITTING IN NEO-GOTHIC ARCHITECTURAL BUILDINGS

Neo-Gothic (also referred to as Victorian Gothic, Gothic Revival or Jigsaw Gothic,) is an architectural movement that began in the late 1740s in England. Its popularity grew rapidly in the early 19th century, when increasingly serious and learned admirers of neo-Gothic styles sought to revive medieval Gothic architecture, in contrast to the neoclassical styles prevalent at the time. Neo-Gothic architecture often has certain features, derived from the original Gothic architecture style, including decorative patterns, finials, scalloping, lancet windows, hood mouldings, and label stops[7]. We now analyse the following structures.

3.1 University of Mumbai Library

Known as the University of Bombay earlier, this institution is one of the oldest institutions in the country, established in 1857. Its architecture is Venetian Gothic inspired [7]. It is possible to take a walk around the campus, and have a peek inside both the University Library and Convocation Hall. The University Library has exquisite stained glass windows that have been restored to pristine glory.

Fig 1 (a) University of Mumbai Library

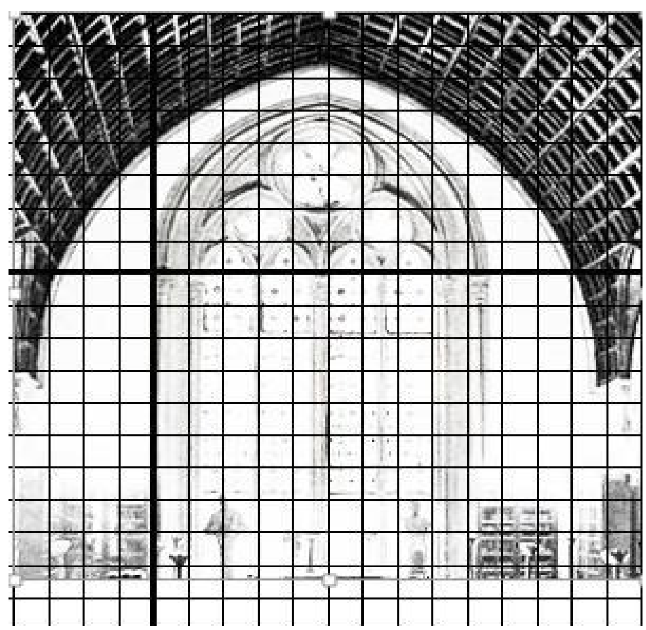

Fig 1(b) Enlarged view of curved lancet window

We shall analyse the shape of the curved lancet window of the library. The data points were taken from Figure 1(b):
(0, 0), (0.5, 1.85), (1, 2), (1.25, 2.35), (2, 2.5), (3, 2.55), (3.5, 2.4), (4, 2.2), (4.5, 1.75) and (4.7, 0)
A MATHEMATICA^{TM} input provided the following:
In:

```
x1=0;x2=0.5;x3=1.0;x4=1.5;x5=2.0;x6=2.5;x7=3;x8=3.5;x9=4;x10=4.5;x11=4.7;
j={0, 1.85, 2, 2.35, 2.5, 2.55, 2.4, 2.2, 1.75, 0.65, 0};
X={{1, x1, x1^2}, {1, x2, x2^2}, {1, x3, x3^2}, {1, x4, x4^2}, {1, x5, x5^2}, {1, x6, x6^2}, {1, x7, x7^2}, {1, x8, x8^2}, {1, x9, x9^2}, {1, x10, x10^2}, {1, x11, x11^2}};
```

B=PseudoInverse[X].j
Norm[X.B]^2/Norm[j]^2
Out:
{0.361709， 2.03832， -0.441087}
0.982978
that is， a parabola with equation y=0.36+2.04x-0.44x^2. It fits well since， R^2 =98.2978%.

If we want an even better approximation， we modify the second co-ordinates slightly as follows:
x1=0;x2=0.5;x3=1.0;x4=1.5;x5=2.0;x6=2.5;x7=3;x8=3.5;x9=4;x10=4.5;x11=4.7;

j={1， 1.85， 2.05， 2.35， 2.5， 2.55， 2.4， 2.2， 1.75， 1.55， 1};

X={{x1^2， y1^2}， {x2^2， y2^2}， {x3^2， y3^2}， …{x11^2， y11^2};

This produces the following outcome: y=-1.12+1.21x-0.26x^2， fitting at 99.74%.

We now plot the corresponding to the initial data points and the modified data graphs using MATHEMATICA™

Data:

{{0， 0}， {0.5， 1.85}， {1， 2}， {1.5， 2.35}， {2， 2.5}， {3， 2.55}， {3.5， 2.4}， {4， 2.2}， {4.5， 1.75}， {4.7， 0}};

y=0.36+2.04x-0.44x^2;

modified=1.12+1.21x-0.26x^2;

Show[ListPlot[Data， PlotStyle□Red]， Plot[{y， modified}， {x， 0， 5}]]

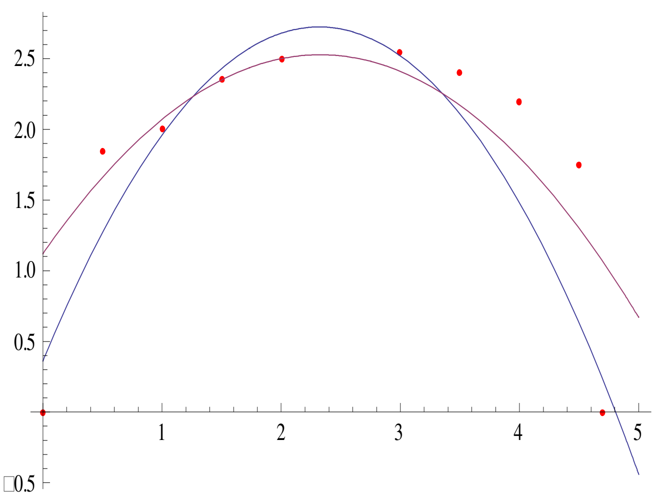

Fig 1(c) The parabolas for the initial data points and modified data points

The initial data points taken on the lancet windows are shown as points. It can be seen that the parabola with the modified data points lie closer to the original data points.

Therefore, it can be seen that if we modify the co-ordinates slightly, we get better approximations mathematically and graphically.

3.2 Santhome Basilica, Chennai

Santhome Basilica is a Roman Catholic minor basilica in Santhome, in the city of Chennai (Madras) India. It was built in the 16th century by Portuguese explorers, over the supposed tomb of St an apostle of Jesus. In 1893, it was rebuilt as a church with the status of a cathedral by the British. The British version still stands today. It was designed in Neo-Gothic style, favoured by British architects in the late 19th century [10].

Fig 2 shows the interior of the Santhome Basilica. Since the arch looks symmetrical, we consider only the points on one side of the line of symmetry.

To get the data points, we first draw the picture with the grid. We then use *Google Sketchup* to compute the various points (x_i, y_i).

The points are:
$x_1 = 0$, $x_2 = 0.32$, $x_3 = 0.64, x_4 = 0.96, x_5 = 1.28, x_6 = 1.6, x_7 = 1.92, x_8 = 2.24, x_9 = 2.56, x_{10} = 2.88, x_{11} = 3.2$, , $x_{12} = 3.52$

$y_1 = 2.50$, $y_2 = 2.48, y_3 = 2.45, y_4 = 2.40, y_5 = 2.3, y_6 = 2.2, y_7 = 2.12, y_8 = 1.96\ y_9 = 1.76, y_{10} = 1.48, y_{11} = 1.3, y_{12} = 0.66$

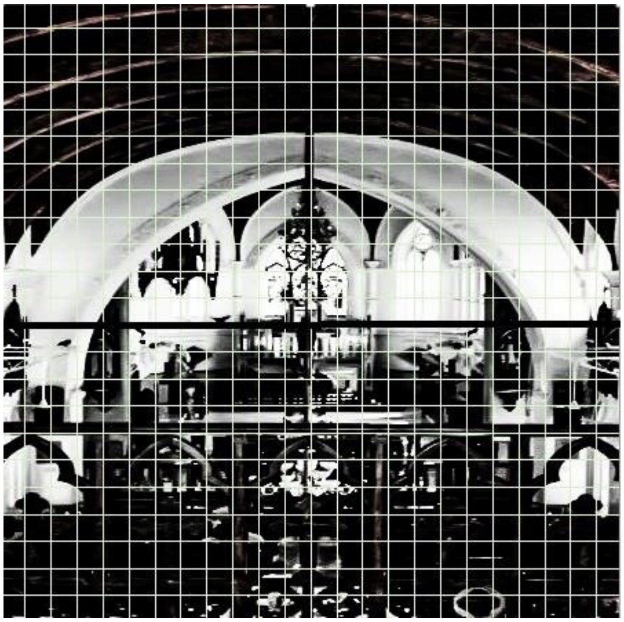

Fig 2 Interior of the Santhome Basilica, Chennai

A MATHEMATICA™ input provided the following:

In:

x1=0;x2=0.32;x3=0.64;x4=0.96;x5=1.28;x6=1.6;x7=1.92;x8=2.24;x9=2.56;x10=2.88;x11=3.2;x12=3.52;

y1=2.5;y2=2.48;y3=2.45;y4=2.3;y5=2.22;y6=2.12;y7=1.96;y8=1.76;y9=1.48;y10=1.12;y11=0.66;y12=0;

X={{x1^2, y1^2}, {x2^2, y2^2}, {x3^2, y3^2}, {x4^2, y4^2}, {x5^2, y5^2}, {x6^2, y6^2}, {x7^2, y7^2}, {x8^2, y8^2}, {x9^2, y9^2}, {x10^2, y10^2}, {x11^2, y11^2}, {x12^2, y12^2}};

j={1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1};
B=PseudoInverse[X].j
Norm[X.B]^2/Norm[j]^2
Out:
{0.0890589, 0.167514}
0.997861

The equation of the ellipse is $\frac{x^2}{0.09}+\frac{y^2}{0.17}=1$
And since, R^2 = 0.998, we see that this ellipse is a 'good' fit.

4. APPLICATION 2: ANALYSIS OF THE SHAPE OF THE COOLING TOWERS AT SIPAT THERMAL POWER PLANT

The Sipat Super Thermal Power Station is located at Sipat in Bilaspur district in state of Chhattisgarh. The power plant is one of the coal based power plants of NTPC [9]. The first unit of the plant was commenced on August 2008. Four induced draft cooling towers are installed at this power plant. Originally, natural draft cooling towers were cylindrical in shape. As the design of these types of towers evolved and the towers were made increasingly larger, the cylindrical shape was changed to hyperbolic, since hyperbolic shape offers superior structural strength and resistance to ambient wind loadings.

We now discuss about the profile of the shape of these towers.

Fig 3 (a) Sipat Thermal Power Plant

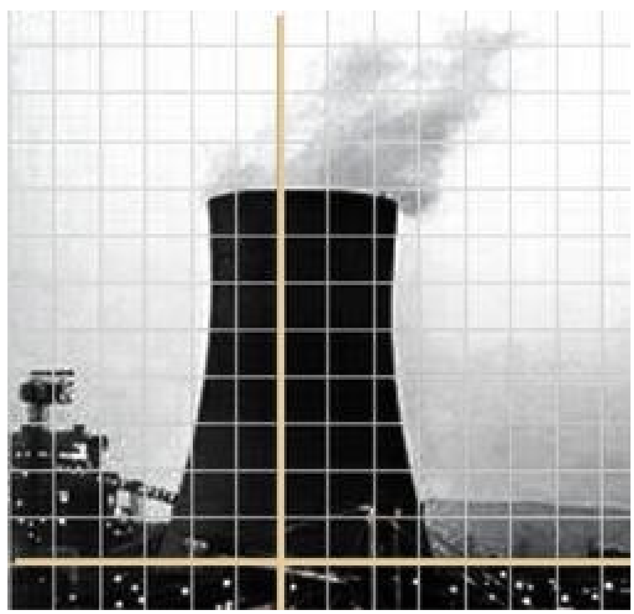

Fig 3 (b) Determining the profile of Thermal Power Plant

Let us consider figure 3(b), from which we take the following values of (x, y):
$x_1 = 1.9$, $x_2 = 1.5$, $x_3 = 1.3, x_4 = 1.2, x_5 = 1.15, x_6 = 1.15, x_7 = 1.2, x_8 = 1.8$
$y_1 = 0$, $y_2 = 0.5, y_3 = 1, y_4 = 1.5, y_5 = 2, y_6 = 2.5, y_7 = 3, y_8 = 3.5$

We first compute the general conic section with equation –

$$Ax^2 + Bxy + Cy^2 + Dx + Ey - 1 = 0$$

The minimal norm least square solution follows from:

x1=1.9;x2=1.5;x3=1.3;x4=1.2;x5=1.15;x6=1.15;x7=1.2;x8=1.25;
y1=0;y2=0.5;y3=1.0;y4=1.5;y5=2.0;y6=2.5;y7=3.0;y8=3.5;
X={{x1^2, x1*y1, y1^2, x1, y1}, {x2^2, x2*y2, y2^2, x2, y2}, {x3^2, x3*y3, y3^2, x3, y3}, {x4^2, x4*y4, y4^2, x4, y4}, {x5^2, x5*y5, y5^2, x5, y5}, {x6^2, x6*y6, y6^2, x6, y6}, {x7^2, x7*y7, y7^2, x7, y7}, {x8^2, x8*y8, y8^2, x8, y8}};
j={1, 1, 1, 1, 1, 1, 1, 1};
B=PseudoInverse[X].j
Norm[X.B]^2/Norm[j]^2

The output turns out to be $-0.17x^2 + 0.07xy - 0.05y^2 - 0.85x + 0.14y = 1$ at 99.9989%
This is the equation of an ellipse since $0.007^2 - 4 * -0.17 * -0.05 < 0$
However, if we lift the x-axis by 2.5 units, we get a hyperbola. This can be seen from the following input in MATHEMATICA™:

x1=1.9;x2=1.5;x3=1.3;x4=1.2;x5=1.15;x6=1.15;x7=1.2;x8=1.25;
y1=-2.5;y2=-2.0;y3=-1.5;y4=-1.0;y5=0.5;y6=0.0;y7=0.5;y8=1.0;X={{x1^2, y1^2}, {x2^2, y2^2}, {x3^2, y3^2}, {x4^2, y4^2}, {x5^2, y5^2}, {x6^2, y6^2}, {x7^2, y7^2}, {x8^2, y8^2}};
j={1, 1, 1, 1, 1, 1, 1, 1};
B=PseudoInverse[X].j
Norm[X.B]^2/Norm[j]^2
Out:
{0.800206, -0.261117}
0.974532

In this case, the closeness of fit is only 97.45%. Hence we observe that the cooling towers at the Sipat Thermal Power Plant are elliptical in shape even though they may appear to be hyperbolic.

5. CONCLUSION

We have analyzed the shape of the various arches in the Neo-Gothic architectural structures and the profile of the thermal plant using Campbell and Meyer's method of curve fitting. However, there are many other methods for curve fitting available in the literature [3, 5]. We can use them to analyse the design of various arches and profiles of structures. These can be taken up in future for study.

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