Advanced Research in Electrical and Electronic Engineering
Print ISSN: 2349-5804; Online ISSN: 2349-5812 Volume 1, Number 4 (2014) pp. 55-58

http://www.krishisanskriti.org/areee.html

Economic Load Dispatch Solution using Interval Gradient Method

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Abstract: **Economic Load Dispatch (ELD) optimization problem is the most important issue which is to be taken into consideration in power systems. The problem of ELD in power systems is to plan the power output for each devoted generator unit in such a way that the operating cost is minimized and simultaneously, matching load demand, power operating limits and maintaining stability. This paper describes an interval based approach for economic dispatch problem of power systems where the gradient search method is employed. The initial value of the Lagrange multiplier for each generator is computed from the generator limits and the resulting equations are solved iteratively using interval mathematics. The proposed method is tested on a three generator power system with and without transmission losses and the obtained results are bounded giving guaranteed global minimum.**

1. INTRODUCTION

The increasing energy demand and decreasing energy resources have necessitated the optimum use of available resources. Today, most of the needed quantity of electrical energy is produced in thermal power plants, where the energy sources to produce mechanical energy applied to move the rotor shaft of generating units is produced by fossil fuels. In a practical power system, the power plants are not located at the same distance from the center of loads and their fuel costs are different [1]. Under normal operating conditions, the generation capacity is more than the total load demand and losses, which leads to scheduling of the generation. Therefore, the objective is to find the real power scheduling of the power plant consisting of generators so as to minimize the operating cost. This is done by minimizing the objective cost function by allowing the generator's power to vary within certain limits to meet the load demand.

The purpose of the Economic Load Dispatch (ELD) problem is to obtain the optimal amount of generated power for the fossil-based generating unit in the system by minimizing the fuel cost and emission level simultaneously, subject to various equality and inequality constraints of the power system [2].

ELD problem has been thought of as a mathematically complex and highly nonlinear optimization problem, especially in larger systems and different techniques have been reported in the literature. Many gradient-based deterministic approaches such as Lagrangian relaxation and linear, nonlinear, and dynamic programming techniques have been applied to find the optimal solution of ELD problem [3]. Some of the population-based methods are genetic algorithm (GA), evolutionary programming-based algorithms (EP), particle swarm optimization (PSO) with its variations, tabu search (TS), and hybrid methods combining two or more metaheuristic algorithms [4]. Among the artificial intelligence methods, Hopfield neural networks [5] have been applied to solve the non-linear ELD problems, but these methods suffer from excessive numerical iterations, resulting in huge computations.

The ELD problem is nonlinear and must be solved iteratively. This paper describes an interval based approach where the gradient search method is employed. A Lagrangian augmented function is first formulated and the initial value of the Lagrange multiplier for each generator is computed from the generator limits. The resulting equations are solved iteratively using interval mathematics which provides all the results in interval form.

The rest of the paper is organized as follows. In section 2 the basics of interval arithmetic and in section 3 ELD problem formulations are given. In section 4 the interval based algorithm is presented. In section 5 the application of the proposed method is presented followed by the conclusions in section 6.

2. INTERVAL ARITHMETIC [6]

Interval arithmetic is an arithmetic defined on sets of intervals, rather than on sets of real numbers. It has been invented by R. E. Moore. The power of interval arithmetic lies in its implementation on computers. In particular, outwardly rounded interval arithmetic allows rigorous enclosures for the ranges of operations and functions. This makes a qualitative difference in scientific computations, since the results are now intervals in which the exact result must lie. It has been used recently for solving ordinary differential equations, linear systems, optimization, etc.
Let
$\boldsymbol{x} = ([a, b] | a \leq b, a, b \in R)$

be a real interval, where a is the infimum (lower endpoint) and b is the supremum (upper endpoint) of $\boldsymbol{x}$. The width of interval is defined as $w(\boldsymbol{x}) = a\text{-}b$. The midpoint of the interval is defined as $m(\boldsymbol{x}) = (a+b)/2$. For an n dimensional interval vector $\boldsymbol{x}^* = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n]$, the midpoint of interval vector $\boldsymbol{x}^*$ is given by $m(\boldsymbol{x}^*) = [m(\boldsymbol{x}_1), m(\boldsymbol{x}_2), \ldots, m(\boldsymbol{x}_n)]$. The width of interval vector is $w(\boldsymbol{x}^*) = [w(\boldsymbol{x}_1), w(\boldsymbol{x}_2), \ldots, w(\boldsymbol{x}_n)]$. A degenerate interval has both its lower and upper endpoints same.

Let $\boldsymbol{x} = [a, b]$ and $\boldsymbol{y} = [c, d]$ be two intervals. Let +, -, * and / denote the operation of addition, subtraction, multiplication and division, respectively. If $\otimes$ denotes any of these operations for the arithmetic of real numbers x and y, then the corresponding operation for arithmetic of interval numbers x and y is

$$\boldsymbol{x} \otimes \boldsymbol{y} = (x \otimes y | x \in \boldsymbol{x}, y \in \boldsymbol{y})$$

The above definition is equivalent to the following rules:

$$\boldsymbol{x} + \boldsymbol{y} = [a + c, b + d]$$

$$\boldsymbol{x} - \boldsymbol{y} = [a - d, b - c]$$

$$\boldsymbol{x} * \boldsymbol{y} = [\min(ac, bc, ad, bd), \max(ac, bc, ad, bd)]]$$

$$\frac{\boldsymbol{x}}{\boldsymbol{y}} = [a, b] * \left[\frac{1}{d}, \frac{1}{c}\right], if\ 0 \notin \boldsymbol{y}$$

An interval function $F(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$ of intervals $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n$ is an interval valued function of one or more variables. $F(x_1, x_2, \ldots, x_n)$ is said to be an interval extension of a real function $f(x_1, x_2, \ldots, x_n)$ if $f(x_1, x_2, \ldots, x_n) \in F(x_1, x_2, \ldots, x_n)$, whenever $x_i \in \boldsymbol{x_i}$ for all $i = 1, 2\ldots\ldots, n$. F is said to be inclusion monotonic if

$$x_i \subset y_i \Rightarrow F(\boldsymbol{x}_1, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}) \subset F(\boldsymbol{y_1}, \boldsymbol{y_2}, \ldots, \boldsymbol{y}_n)$$

Also $F(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n})$ contains the range of $f(x_1, x_2, \ldots, x_n)$. Interval functions $F(\boldsymbol{x})$ can be constructed in any programming language in which interval arithmetic is simulated or implemented via natural interval extensions. However, computing an interval bound carries a cost of 2 to 4 times as much effort as evaluating $f(x)$ [6, 7].

3. ELD PROBLEM FORMULATION

The main objective of the ELD problem is to determine minimum generation cost of the generating units, according to the operating constraints of the generators and the power system limits. The thermal power plant can be expressed as input-output models (i.e. cost function), where the input is the fuel cost and the output is the power output of each generating unit. The fuel cost function of each generating unit is approximately represented by a quadratic function. The cost depends on the number of hours the plant is in operation or upon the number of units of electrical energy generated [8].

4. COST FUNCTION

The ELD problem can be mathematically described as follows

$$\mathrm{Min}_{P_i} \sum_{i=1}^{N} F_i(P_i) = \mathrm{Min}_{P_i} \sum_i (a_i + b_i P_i + c_i P_i^2) \quad (1)$$

where, P_i is the power output and a_i, b_i, c_i are the fuel cost coefficients of the i^{th} generating unit.

5. POWER BALANCE CONSTRAINTS

The total electric power generation $\sum_{i=1}^{N} P_i$ must cover the total power demand P_D and the real power loss P_{loss} in the transmission line.

$$\sum_{i=1}^{N} P_i = P_D + P_{loss} \quad (2)$$

If the test system is considered to be lossless, the total electric power generation $\sum_{i=1}^{N} P_i$ equals the power demand P_D. Thus,

$$\sum_{i=1}^{N} P_i - P_D = 0 \quad (3)$$

For all the practical applications the equality constraints are modified as

$$\sum_{i=1}^{N} P_i - P_D = \varepsilon \quad (4)$$

$$\sum_{i=1}^{N} P_i - (P_D - P_{loss}) = \varepsilon \quad (5)$$

where, ε is the equality constraint tolerance taken as 10^{-3}.

6. GENERATION CAPACITY CONSTRAINT

Each generating unit is constrained by its lower and upper limits of real power output to ensure stable operation.

$$P_i^{min} \leq P_i \leq P_i^{max},\ i = 1, 2, \ldots,\ N \quad (6)$$

where, $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum real power output of i^{th} generator, respectively.

A total transmission network loss is a quadratic function of power outputs that can be represented using B coefficients. The approximate loss formula is expressed as

$$P_{loss} = \sum_i^N B_{ii} P_i^2 \quad (7)$$

where B_{ii} are loss coefficients and are taken as constants.

7. INTERVAL GRADIENT METHOD

In the conventional gradient method the constraints are augmented into the cost or objective function by using the Lagrange multipliers.

$$\mathcal{L} = \sum_{i=1}^N F_i(P_i) + \lambda(P_D + P_{loss} - \sum_{i=1}^N P_i) \quad (8)$$

where, λ is the Lagrange multiplier. The minimum of the unconstrained function (8) is found at the point where the partial derivatives w.r.t P_i and w.r.t λ respectively are zero.

Therefore, at any iteration k the condition for optimal dispatch for a lossless system and a system with losses are given by (9) and (10) respectively

$$\lambda_i^{(k)} = b_i + 2c_i P_i^{(k)} \quad (9)$$

$$\lambda_i^{(k)} = \frac{b_i + 2c_i P_i^{(k)}}{1 - 2\sum_{i=}^N B_{ii} P_i^{(k)}} \quad (10)$$

In the proposed method we take the generator limits into consideration by representing P_i as an interval

$$P_i = [P_i^{min}, P_i^{max}]$$

Taking P_i as interval, we obtain the bounded values of λ_i using (9) or (10) for each generator. We take the midpoints of each of them and take their average as λ. Using this value of λ we divide each λ_i into two intervals λ_{i1} and λ_{i2} and compute the interval values of P_{i1} and P_{i2} over both of them for $i=1, .., N$ using (11) and (12) respectively for the two systems.

$$P_{ij}^{(k)} = \frac{\lambda_i^{(k)} - b_i}{2c_i} \quad j = 1, 2 \quad (11)$$

$$P_{ij}^{(k)} = \frac{\lambda_i^{(k)} - b_i}{2(c_i + \lambda_i^{(k)} B_{ii})} \quad j = 1, 2 \quad (12)$$

Next we compute the error over both as given by (13) and (14) for the two systems respectively.

$$\Delta P_j^{(k)} = P_D - \sum_{i=1}^N P_{ij}^{(k)} \quad j = 1, 2 \quad (13)$$

$$\Delta P_j^{(k)} = P_D + P_{loss}^{(k)} - \sum_{i=1}^N P_{ij}^{(k)} \quad j = 1, 2 \quad (14)$$

In (14) $P_{loss}^{(k)}$ is computed using (7) with the k^{th} value of $P_i^{(k)}$. Now we choose that value of $\lambda_{i1}^{(k)}$ or $\lambda_{i2}^{(k)}$ corresponding to the interval $P_{i1}^{(k)}$ or $P_{i2}^{(k)}$ that contains a zero. The same procedure is continued till the error is within a specified tolerance. In the process if any $P_i^{(k)}$ goes beyond its generation limits then it is fixed at its upper or lower end point as the case may be.

Finally using the bounded values of P_i's we obtain the minimum value of the objective function in bounded form.

8. APPLICATION EXAMPLES AND RESULTS

In this paper, the proposed Interval Gradient algorithm is developed in MATLAB 6.1 using the INTLAB toolbox [9]. The effectiveness of the proposed approach is investigated by considering a standard 3-generator system as a test system taken from [1]. The values of fuel cost coefficients, generator limits and the loss coefficients are given in Table 1. The power demand is taken as 150 MW.

Table 1 Fuel Cost Coefficients and Generator unit limits

Gen unit	Fuel cost coefficients			Loss coefficient	Generating unit limits	
i	a_i	b_i	c_i	B_{ii}	P_i^{min}	P_i^{axn}
1	200	7.0	0.008	0.000218	10	85
2	180	6.3	0.009	0.000228	10	80
3	140	6.8	0.007	0.000179	10	70

Initially the system is considered lossless. Therefore, the problem constraints are the power balance constraint without P_{loss} and the generation capacity constraint. In the next case the system with losses is considered. For all the cases we took $\boldsymbol{\varepsilon = 0.001}$.The results obtained in both the cases are shown in Table 2.

Table 2 Results obtained by the proposed method

Output Powers	Lossless system	Lossy system
P_1 MW	[31.9370, 1.9377]	[35.0968, 35.3468]
P_2 MW	[67.2774, 7.2779]	[64.1341, 64.3621]
P_3 MW	[50.7852, 0.7859]	[52.4827, 52.7725]
λ	7.5110	7.6815
Loss MW		[1.6993, 1.7154]
Fuel cost §/hour	[1579.6, 1579.8]	[1592.7, 1598.6]

From Table 2 we can see that the results obtained are in bounded form and contain the results obtained from the conventional gradient method.

9. CONCLUSIONS

In this paper a new method for solving the economic load problem has been proposed. This iterative method is a modified version of the conventional gradient method, where the value of Lagrange multiplier is not to be assumed but have to be computed using the generator limits. All the further computations are done using the concepts of interval analysis. This proposed method gives better results as they are all bounded and hence are verified.

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