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Use of Adomian and Restarted Adomian Methods for Solving Algebraic Equations

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Abstract: **Nonlinear algebraic equations arise frequently in the modeling of many engineering systems. More specifically, these equations appear while mathematically representing the steady state model of a lumped parameter system, e.g. a continuous stirred tank reactor, single or multiple effect evaporators, flash drum etc. In this study, the application of two efficient methods namely, Adomian Decomposition Method (ADM) and one of its variant namely, Restarted ADM (RADM) has been demonstrated by solving an algebraic equation. Some peculiar characteristics of the obtained solutions, depicting the advantages and limitations of these methods, are shown and discussed, which may be followed while solving nonlinear algebraic equations.**

1. INTRODUCTION

Nonlinear algebraic equations (NLAEs) are frequently encountered in the steady state modeling of many important engineering systems/processes e.g. equation of state, sequence of evaporators, various heat and mass transfer equipment etc. In general, different numerical techniques are applied for their solutions e.g. Newton-Raphson method (NRM) and its variants and Broyden class of methods etc, since analytical solutions are not always possible or are very difficult even for the equations in single variable with mild nonlinearity [1-3].

Recently, Adomian decomposition method (ADM) [4, 5] and its modified versions have been successfully employed in a variety of problems. ADM is a effective analytical/numerical method for solving different types of nonlinear equations (algebraic, transcendental, ordinary and partial differential equations etc) having weak or strong nonlinearities [4, 5]. Besides, with the help of mathematical softwares e.g. Mathematica, Maple etc it can easily be programmed.

Numerical solution of nonlinear algebraic and transcendental equations using ADM was first demonstrated by Adomian himself in his original work [4]. Thereafter, several workers studied the theoretical and practical aspects of ADM and its various versions [6-12]. The present work concerns with the application of ADM and one of its variant namely, RADM to obtain the numerical solution of a polynomial equation. To the best of author's knowledge such analysis of ADM/RADM is unavailable in literature, and thus this study may provide more insight while solving nonlinear algebraic equations using ADM/RADM.

2. SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS BY ADM

The basic philosophy of ADM is to break the nonlinear equation into a set of infinite but simpler linear equations by decomposing the nonlinearity into components called Adomian polynomials, A_i. These polynomials can easily be generated for any type of nonlinearity $\square(y)$ with the help of several effective algorithms [4] and the obtained linear equations are solved successively to get the final solution. Ultimately, the analytical solution is obtained in the form of an infinite series called Adomian series, which is basically a generalized Taylor series around a function rather than around a point and converges more rapidly than the Taylor series [12]. The application of ADM for the solution of nonlinear algebraic equations is explained below.

Consider a nonlinear algebraic equation in single variable i.e. $\square(y) = 0$. Expressing this equation in the following canonical form [4]:

$$y = C_0 + F_0(y) \tag{1}$$

Where, C_0 is a constant and $F_0(y)$ is some function of y. Now, y and $F_0(y)$ are decomposed as follows [4]:

$$y = \sum_{i=0}^{\infty} y_i = C_0 + \sum_{i=0}^{\infty} A_i \tag{2}$$

Where, y_i s are the decomposed solutions i.e. $y_i = \frac{1}{i!}\frac{\partial^i y}{\partial \lambda^i}\bigg|_{\lambda=0}$

and A_i s are Adomian polynomials i.e.

$A_i = \frac{1}{i!}\frac{\partial^i F_0(y)}{\partial \lambda^i}\bigg|_{\lambda=0} = \frac{1}{i!}\frac{\partial^i}{\partial \lambda^i} F_0(\sum_{i=0}^{\infty} y_i)\bigg|_{\lambda=0}$. Comparison of terms in Eq. (2) leads one to obtain $y_0 = C_0$, and $y_i = A_{i-1}(y_0, y_1, ..., y_{i-1})$ and eventually, the solution y becomes known. Convergence related issues for the above Adomian method are given in Ref. [12]. It is to be noted that this procedure can also be extended, in a similar way, to obtain the solutions of coupled NLAEs involving more than one variable [13, 14].

Since, there are many ways in which the canonical form i.e. Eq. (1) can be expressed and because, the form of final series solution strongly depends on it therefore, the convergence of the solution series may differ appreciably. However, if the series is convergent it will converge to the smallest magnitude root, as proved by Adomian [4], and the other roots can be found by the deflation method.

3. SOLUTION OF POLYNOMIAL EQUATION BY ADM AND RADM

Solution of polynomial equations using ADM

Consider the following nth degree polynomial equation:

$$a_n y^n + ... + a_1 y + a_0 = 0,\ n > 1, a_i \in \mathbb{R},\ \&\ a_0 \neq 0 \qquad (3a)$$

Expressing Eq. (3a) in the following canonical form:

$$y = \underbrace{-\frac{a_0}{a_1}}_{C_0} - \underbrace{\left(\frac{a_2}{a_1} y^2 + ... + \frac{a_{n-1}}{a_1} y^{n-1} + \frac{a_n}{a_1} y^n\right)}_{F_0(y)} \qquad (3b)$$

Thereafter, ADM is applied to solve the above Eq. (3b) and the solution y is obtained, provided $a_0 \neq 0$. In case $a_0 = 0$, a constant may be added in place of C_0 and subtracted from $F_0(y)$, afterward the resultant equation may be subjected to ADM. To get more insight, the solutions of a 3rd degree polynomial equation have been obtained by using ADM for various values of a_3, a_2, a_1, a_0, and the results are shown in Table 1. Table 1 highlights some of the instances where ADM failed and thus limits the scope of ADM for such cases. To overcome these limitations, some other modified versions of ADM have been proposed [13-17], out of which a very effective variant of ADM i.e. RADM has been described in the following subsection.

Solution of polynomial equations using RADM

As seen previously in the treatment of 3rd degree polynomial equation, the ADM may have slow convergence or may even diverge. To avoid these problems, Babolian and Biazar [16] and Babolian and Javadi [17] modified the ADM naming it Restarted ADM (RADM) to get the results more accurately and quickly. Though, RADM is not fool proof but many a times it performed better than ADM. A brief description of RADM is given below and the details can be found elsewhere [16, 17].

Eq. (1) is updated using the following procedure:

Step 1: $C_{i+1} = y^{(i)}$ (4a)

Step 2: $F_{i+1}(y) = \frac{F_0(y) - yF_0'(C_{i+1}) + C_0}{1 - F_0'(C_{i+1})} - C_{i+1}$ (4b)

$F_0'(C_{i+1}) \neq 1$

Step 3: $y^{(i+1)} = C_{i+1} + F_{i+1}\left(C_{i+1}\right), i = 0,1,2...$ (4c)

The C_0 and $F(y)$ in Eq. (1) are updated using above steps and the solution is obtained by applying ADM to the updated equation i.e. Eq. (4c). The value of solution so found is used to update C_{i+1} through Eq. (4a); initially, C_1 is taken equal to C_0. This course of action is repeated until one gets the result of desired accuracy.

To check the effectiveness of RADM the previous set of cubic equations has again been solved by it and the results are also presented in Table 1. It is observed that RADM successfully found the solutions in a few iterations except where canonical form cannot be formed. After performing many careful numerical experiments on various nonlinear algebraic and transcendental equations some guide lines have been formulated. In case of slow convergence or failure of the ADM one can opt either or the combinations of the following:
Other canonical forms of equation may be experimented with.
(iia) Use the RADM.
(iib) If RADM also fails, either follow suggestion (i) or add and subtract a constant in Eq. (1) so as to modify C_0 and $F_0(y)$ as advised in Ref. [18].

In our experience, steps (iia) and (iib) are best to follow as almost all the problems can be handled by these steps. It is worth noting that the above two approaches (ADM and RADM) as well as these two steps are also applicable to other NLAEs and not just limited to polynomial equations.

Analysis of the above ADM/RADM results, obtained for different canonical forms, can be summarized as follows:

i) Both the methods are sensitive towards the canonical form of an equation.
ii) Increasing the terms in both the methods does not always ensure the convergence.
iii) In general, RADM is superior to ADM and incase of breakdown of RADM, the proposed guidelines are very helpful.

Table 1. Solutions of cubic equations

a_3, a_2, a_1, a_0	Smallest abs root	Results using ADM	Results using RADM
1, -6, 11, -6	1	0.849439 (n_T =5), 0.997553 (n_T =50)	0.918895 (m=1) 0.998406 (2) 1.00 (3)
1, -11/6, 1, -1/6	1/3	0.264382 (n_T =5), 0.328324 (n_T =50)	0.289849 (1) 0.329913 (2) 0.333329 (3) 0.333333 (4)
1, 6, 11, 6	-1	-0.849439 (n_T =5), -0.997553 (n_T =50)	-0.918895 (1) -0.998406 (2) -0.999999 (3) -1.00 (4)
1, 11/6, 1, 1/6	-1/3	-0.264382(n_T =5), -0.328324 (n_T =50)	-0.289849 (1) -0.329913 (2) -0.333329 (3) -0.333333 (4)
1, 4, 1, -6	1	ADM failed as series diverged.	3.04703 (1) 1.56364 (2) 1.04731 (3) 1.00007 (4) 1.00000 (5)
1, -1/6, -2/3, -1/6	-1/3	-0.318742(n_T =5), -0.333321(n_T =50)	-0.326532 (1) -0.333318 (2) -0.333333 (3)
1, 0, -7, 6	1	0.993045(n_T =5), 0.999999 (n_T =50)	0.998166 (1) 0.999999 (2) 1.00 (3)
1, -7/6, 0, 1/6	-1/3	ADM cannot be used as canonical form cannot be formed ($a_1 = 0$).	RADM cannot be used as canonical form cannot be formed ($a_1 = 0$).
1, -5/4, 1/8, 1/8	-1/4	ADM failed as series diverged.	-0.490489 (1) -0.283132 (2) -0.250259 (3) -0.250000 (4)

4. CONCLUSIONS AND RECOMMENDATIONS

The application of ADM and RADM was demonstrated for obtaining the numerical solution of algebraic equations. It was observed that the convergence of solutions depends on several factors namely equation form, number of terms, the method employed and the parameters' ranges. In most cases, it was found that RADM outperformed ADM but in exceptional cases ADM can converge whilst RADM may diverge. Nevertheless, RADM is an efficient method and its effectiveness can be increased further by properly following the proposed guidelines.

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