Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 1, Number 10; October, 2014 pp. 85-88

http://www.krishisanskriti.org/jbaer.html

Nonlinear Electron Acoustic Waves via non Thermal Electron Distribution in Plasma

Sona Bansal

Sikh National College, Banga-144505(Punjab),
sonabansal@yahoo.com

Abstract: **The effects of non thermal electron distribution on finite amplitude non linear electron acoustic waves is studied in an collisonless homogenous and unmagnetized plasma which consist of cold and hot electrons as well as ions. A nonlinear Schrodinger equation is derived to study the modulational instability of finite amplitude electronacoustic waves by using the standard reductive perturbation technique. It is found that the presence of nonthermally distributed electrons modifies the domain of the modulational instability and solitary structures. Possibility of stationary states of the wave packets that can appear as envelope solitons under different conditions is explored. The present investigation is relevant to observation from the Viking satellite in the dayside auroral zone.**

Keywords: **Waves, oscillations and instabilities in plasma and intense beams**

1. INTRODUCTION

Electron-acoustic wave (EAW) is an electrostatic (ES) wave, which had been first discovered experimentally [1-2]. It is a high frequency mode in plasma where a minority of inertial cold electrons oscillate against a dominant background of inertia less hot electrons. The latter provides the necessary restoring force while ion dynamics does not play any role except charge neutralization. The two populations are often referred to cold (hot) electrons with respective temperatures $T_c(T_h)$ and such plasma support electrostatic electron acoustic wave (EAW) as its natural mode. The propagation of electron-acoustic wave (EAW) is only possible within a restricted range of parameter values. A more rigorous analysis (Tokar and Gary (1984); Gary and Tokar (1985); Mace and Hellberg (1990); Mace et al. (1999)) shows that EAW will e heavily damped unless $T_c << T_h$ and cold electrons represent a significant fraction of plasma .The plasmas with different temperatures and masses often occur in laboratory (Derfler and Simonen (1969). The wave dynamics of the EAWs has received a great deal of renewed interest because of its potential relevance in interpreting electrostatic component of the broadband electrostatic noise (BEN) observed in the cusp of terrestrial magnetosphere (Tokar and Gary (1984)), in geomagnetic tail(Shriver and Asour-Abdalla (1989)), in Auroral region, in the Earths bow shock, the heliospheric termination of shock as well as planetary and neutron star magnetospheres[4-7]. Satellite based observations provided abundant information that the localized electrostatic structures are of nonlinear type and solitary waves in plasma sheet boundary are the electron waves, possibly an electron acoustic solitary wave (EASW). Several studies on the nonlinear electron acoustic waves have been reported in the past (Buti (1980); Buti et al. (1980); Yu and Shukla (1983); Tagare et al. (2004)). From theoretical point of view, the large amplitude nonlinear waves for understanding BEN, were pointed by Mace et al. (1991). Dubouloz et al. (1991;1993) rigorously studied the BEN observed in the dayside of Auroral zone and explained duration burst of BEN in terms of EA solitary waves. Berthomier et al. (2000) pointed out that the positive potential structures are very important from the point of view of the interpretation of various ES structures observed in the auroral region at intermediate altitude by FAST, at higher altitude by POLAR and in geomagnetic tail by GEOTAIL[8-12]. Berthomier et al. (2000) and Singh et al. (2001) have studied the electronacoustic solitons in four component plasmas.

In most of the research publications mentioned above, hot electron component in two-electron temperature plasma was assumed to follow Maxwellian distribution. However, some recent observations show that the particles may not follow Maxwellian distribution. Such particle distributions are called non-Maxwellian type and based on the data, particle distributions are better modelled by velocity distributions having flat top with high energy tails[3]. These distributions, for example, nonthermal particles distributions have abundance of superthermal particles. Another type of non-Maxwellian distribution extensively used in several investigations, is nonisothermal velocity distribution, resulting from the formation of phase space holes. Cairns et al. (1995a;1995b) were the first to explain the structure of solitary waves with density depression using nonthermal distribution for electrons. It has also been noticed that electron and ion distributions play a crucial role in characterizing the physics of the nonlinear waves and this motivated many researchers to study EASWs in a variety of plasma environments. Among the best known paradigms used to study nonlinear behaviour are Korteweg-de-Vries (KdV) equation and its variants, or nonlinear Schrodinger equation (NLSE). Some form of reductive perturbation theory (RPT) is used to derive these equations. The KdV equation describes the evolution of the unmodulated wave and the bare pulse does not contain high frequency oscillations inside the packet. This special solution is also called KdV soliton, in which dispersion is compensated by the nonlinearity. On the other hand, NLSE governs the dynamics of a modulated wave packet. Here, the nonlinearities are in balance with wave group dispersion and the resulting solutions of NLSE possess envelope structures, known as envelope solitons[4].

For envelope soliton, there has been an increased interest in recent years on the investigation of modulational instability of different wave modes in plasma because of its importance in stable wave propagation. However, only a few investigations are reported for ion-acoustic mode [5-11]. It is further observed that EA waves being high frequency density waves, are trapped and modulated leading to modulation and generation of electron-acoustic envelope solitons. In high time resolution of the FAST observations, these kinds of nonlinear structures are observed [12]. Most of the investigations reported so far, have been restricted to modulational instability of ion-acoustic waves in plasma with two temperature electrons [7]. In the present investigation, we study the modulational in-stability of EA waves in plasma with nonthermal electrons. We have used the range of parameters of auroral zone plasma measured from Viking satellite [10,13]. Using the reductive perturbation technique, we have derived the NLSE, which governs the slow modulation of the wave amplitude. In Section 2, we have introduced basic equations governing the dynamics of EA mode and derived the nonlinear Schrdinger equation using reductive perturbation method. Stability analysis and discussion of the results are presented in the last section.

2. BASIC EQUATION

Since the plasma with two electron populations do occur frequently in laboratory and space, EA waves play an important role in such environments. We consider a collisionless infinite homogeneous and Unmagnetized plasma in a following model. The plasma fluid model consists of cold and hot electron components referred to here c and h respectively. The presence of two such population groups are necessary for the existence of EA wave (Tokar and Gary (1984); Gary and Tokar (1985)). The dimensionless fluid equations governing the dynamics of electron acoustic wave are given as follows:

$$\frac{\partial n_c}{\partial t}+\frac{\partial (n_c u_c)}{\partial x}=0 \quad (1)$$

$$\frac{\partial u_c}{\partial t}+u_c\frac{\partial u_c}{\partial x}+\frac{3\alpha(1+\alpha)^2 n_c}{\theta}\frac{\partial n_c}{\partial x}-\alpha\frac{\partial \phi}{\partial x}=0 \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2}=\frac{1}{\alpha}n_c+n_h-\left(1+\frac{1}{\alpha}\right) \quad (3)$$

where $n_h=(1-\beta\phi+\beta\phi^2)e^{\phi}, \theta=\frac{T_h}{T_c}$ and $\alpha=\frac{n_{h0}}{n_{c0}}, n_c$ and n_h are the normalized number densities of cold and hot electrons, respectively. ϕ is the normalized electrostatic potential, u_c is the normalized velocities of cold electron in the x direction. x and t are also normalized. The normalization is as follows.

The densities of cold and hot electrons are normalized by n_{co} and n_{ho}. The space coordinates x time t, velocity and electrostatic potential ϕ are normalized by the hot electron Debye length $(K_B T_h/4\pi\, n_{ho} e^2)^{1/2}$, inverse of cold electron plasma frequency $\omega_{pc}^{-1}=\sqrt{(m/4\pi\, n_{co} e^2)}$, $c_e=\sqrt{(K_B T_h/\alpha m)}$, and $\frac{K_B T_h}{e}$, respectively. Here m is the electron mass, e is the magnitude of the electron charge and K_B is the Boltzmann constant. If we assume that the wave packet moves with the group velocity, which is determined by the linear dispersion relation of equations (1) - (3), then we introduce the following stretched variables by using the standard reductive perturbation method, as used by Taniuti and Yajima (1969):

$$\xi=\epsilon(x-v_g t), \quad (4)$$

$$\tau=\epsilon^2 t, \quad (5)$$

where v_g is the group velocity to be determined by the compatibility requirement. ϵ is a small formal expansion parameter and is the measure of perturbation. The condition $\epsilon \ll 1$ implies that the plasma dimensions must be much larger than the Debye length, which is satisfied in most cases of interest. We will assume that all perturbed quantities depend on the fast scale via the phase $\xi=kx-\omega t$ only, while the slow scales enter the argument of the lth harmonic amplitude, say for density as n_l^n. Following this prescription, the dependent variables are expanded as:

$$n_c = 1+\sum_{n=1}^{\infty}\sum_{l=-\infty}^{\infty}\epsilon^n\, n_l^n(\xi,\tau)\, e^{\iota l(kx-\omega t)} \quad (6)$$

$$u_c = \sum_{n=1}^{\infty}\sum_{l=-\infty}^{\infty}\epsilon^n\, u_l^n(\xi,\tau)\, e^{\iota l(kx-\omega t)} \quad (7)$$

$$\phi = 1+\sum_{n=1}^{\infty}\sum_{l=-\infty}^{\infty}\epsilon^n\, \phi_l^n(\xi,\tau)\, e^{\iota l(kx-\omega t)} \quad (8)$$

where n_c, u_c, ϕ satisfy the reality condition $A_{-l}^{(n)}=A_l^{(n)*}$ and asterisk denote the complex conjugate. Using (4) through (8) in (1) to (3) and collecting the terms of different powers of ϵ, we get the reduced equations. For the first order (n = 1), we get

$$-\iota\omega n_1^{(1)}+\iota k u_1^{(1)}=0 \quad (9)$$

$$-\iota\omega u_1^{(1)}+\frac{3\iota k\alpha}{\theta}(1+\alpha)^2 n_1^{(1)}-\iota k\alpha\phi_1^{(1)}=0 \quad (10)$$

$$(1-\beta+k^2)\phi_1^{(1)}+\frac{1}{\alpha}n_1^{(1)}=0 \quad (11)$$

Algebraic manipulations of these equations lead to the following dispersion relation

$$\omega^2=\frac{k^2}{1-\beta+k^2}+\frac{3k^2\alpha(1+\alpha)^2}{\theta} \quad (12)$$

From (9) to (11), we can express the first order quantities in terms of $\phi_1^{(1)}$ as

$$n_1^{(1)}=-\alpha(1-\beta+k^2)\phi_1^{(1)} \quad (13)$$

$$u_1^{(1)}=-\alpha\frac{\omega}{k}(1-\beta+k^2)\phi_1^{(1)} \quad (14)$$

For the second order (n = 2), reduced equation with l = 1, we get

$$-\iota\omega n_1^{(2)}+\iota k u_1^{(2)}=v_g\frac{\partial n_1^{(1)}}{\partial\xi}-\frac{\partial u_1^{(1)}}{\partial\xi} \quad (15)$$

$$-\iota\omega u_1^{(2)}+\frac{3\iota k\alpha}{\theta}(1+\alpha)^2n_1^{(2)}-\iota k\alpha\phi_1^{(2)}=v_g\frac{\partial u_1^{(1)}}{\partial\xi}+\frac{3\iota k\alpha}{\theta}(1+\alpha)^2\frac{\partial n_1^{(1)}}{\partial\xi}+\alpha\frac{\partial\phi_1^{(1)}}{\partial\xi} \quad (16)$$

$$(1-\beta+k^2)\phi_1^{(2)}+\frac{1}{\alpha}n_1^{(2)}=2\iota k\frac{\partial\phi_1^{(1)}}{\partial\xi} \quad (17)$$

Using (13) to (17), we can put the second order quantities n(2) 1 ,u(2) 1 in terms of $\phi_1^{(2)}$ and $\frac{\partial\phi_1^{(1)}}{\partial\xi}$. Then, these are further algebraically manipulated and we obtain the following compatibility condition:

$$v_g=\frac{k}{\omega}\left[\frac{1-\beta}{(1-\beta+k^2)^2}+\frac{3\alpha(1+\alpha)^2}{\theta}\right] \quad (18)$$

The second harmonic mode of the carrier, which comes from nonlinear self-interaction, is also obtained in terms of$\left[\phi_1^{(1)}\right]^2$. The component l = 2 for the second order, n=2, reduced equations determine the second order quantities. They turn out to be

$$u_2^{(2)}=D\left[\phi_1^{(1)}\right]^2 \quad (19)$$

$$n_2^{(2)}=\left[\frac{k}{\omega}D+(1-\beta+k^2)^2\alpha^2\right]\left[\phi_1^{(1)}\right]^2 \quad (20)$$

$$\phi_2^{(2)}=\frac{\left[-\frac{2}{\alpha}\left(\frac{k}{\omega}D+\left(1-\beta+k^2\right)^2\alpha^2\right)-1\right]}{2(1-\beta+4k^2)}\left[\phi_1^{(1)}\right]^2 \quad (21)$$

where

$$D=\frac{\alpha\omega(1-\beta+k^2)}{6k^3}\Big[\alpha(1-\beta+k^2)(1-\beta+4k^2)+\frac{12\alpha^2}{\theta}(1+\alpha)(1-\beta+k^2)(1-\beta+4k^2)+2\alpha(1-\beta+k^2)^2+1\Big] \quad (22)$$

The nonlinear self-interaction of the carrier wave also leads to the creation of a zeroth order harmonic. Its strength is analytically determined by taking l = 0 component of the third order reduced equations i.e., for n = 2 , l = 0. The result is expressed in terms of the square of modulus of n=1, l=1 i.e., $\left[\phi_1^{(1)}\right]^2=\phi_1^{(1)}\phi_1^{(1)*}$

$$n_0^{(2)}=B\left[\phi_1^{(1)}\right]^2 \quad (23)$$

$$\phi_0^{(2)}=-\left(\frac{1}{1-\beta}\right)\left(\frac{B}{\alpha}+1\right)^2\left[\phi_1^{(1)}\right]^2 \quad (24)$$

$$u_0^{(2)}=\left(Bv_g-\frac{2\omega}{k}(1-\beta+k^2)^2\alpha^2\right)\left[\phi_1^{(1)}\right]^2 \quad (25)$$

where

$$B=\frac{-\alpha\left[\frac{2\alpha}{k}(1-\beta)(1-\beta+k^2)^2\left(\omega v_g+\frac{3\alpha(1+\alpha)^2k}{\theta}\right)+\alpha(1-\beta+k^2)(1-\beta)+1\right]}{\left[1-(1-\beta)\left(v_g{}^2-\frac{3\alpha(1+\alpha)^2k}{\theta}\right)\right]} \quad (26)$$

Finally, substituting the above derived expressions into l = 1 component of the third order (n=3) part of the reduced equation, we obtain the following nonlinear Schrodinger equation (NLSE):

$$\iota\frac{\partial\phi}{\partial\tau}+P\frac{\partial^2\phi}{\partial\xi^2}+Q|\phi|^2\phi=0 \quad (27)$$

where

$$P=-\frac{3}{2}\frac{k^4}{\omega^3(1-\beta+k^2)^4}\left[(1-\beta)+\left(\frac{3\alpha(1+\alpha)(1-\beta)}{\theta}-\frac{k^2\alpha(1+\alpha)}{\theta}\right)(1-\beta+k^2)\right] \quad (28)$$

$$Q=\frac{k}{2\alpha\omega^2(1-\beta+k^2)}\Bigg[-Bk\omega\left(\alpha+\frac{1}{(1-\beta+k^2)(1-\beta)}+\frac{2\alpha(1-\beta)}{(1-\beta+k^2)}+\frac{12\alpha^2(1-\beta+k^2)(1+\alpha)}{\theta}\right)-k^2D\left[\frac{1}{(1-\beta+k^2)(1-\beta+4k^2)}+3\alpha\left(1+\frac{4\alpha(1-\beta+k^2)(1+\alpha)}{\theta}\right)\right]+3\omega(1-\beta+k^2)^2k\left(1+\frac{2\alpha(1-\beta+k^2)(1+\alpha)}{\theta}\right)+\frac{\omega k\alpha(1+3\beta)}{2(1-\beta+k^2)}-\frac{k\alpha\omega}{(1-\beta+k^2)(1-\beta)}-\frac{k\omega\left(2\alpha\left(1-\beta+k^2\right)^2-1\right)}{2(1-\beta+k^2)(1-\beta+4k^2)}\Bigg] \quad (29)$$

In the NLSE (27), we have replaced $\phi_1^{(1)}$ by ϕ for the sake of notational convenience.

3. STABILITY ANALYSIS AND DISCUSSION

In the standard stability analysis, we linearize around the monochromatic wave solution of the NLSE and modulation on the wave amplitude takes place in the propagation direction. Therefore, we separate the amplitude ϕ into two parts as follows:

$$\phi=[\phi_0+\delta\phi(\zeta)]e^{(-\iota\Delta\tau)} \quad (30)$$

where $\zeta = k\xi - \Omega\tau$ is the modulational phase and 0 < K << k and Ω << ω are respectively the wave number and frequency of modulation. ϕ_0 is the amplitude of pump carrier wave, $\delta\phi$ $<<\phi_0$ small amplitude perturbation and Δ is a nonlinear frequency shift. Substituting (31) into (27) and collecting the terms of same order, we obtain

$$\Delta = -Q|\Phi_0|^2 \quad (31)$$

and

$$\iota\frac{\partial\phi}{\partial\tau} + P\,\frac{\partial^2\phi}{\partial\xi^2} + Q|\Phi_0|^2\,(\delta\phi + \delta\phi^*) = 0 \quad (32)$$

where $\delta\phi^*$ is complex conjugate of $\delta\phi$. On assuming that the amplitude perturbation varies as exp $[\iota(k\xi - \Omega\tau]$and following the standard procedure [30], after simplification we get

$$\Omega^2 = PK^2(PK^2 - 2Q|\phi_0|^2) \quad (33)$$

Equation (33) is nonlinear dispersion relation for the amplitude modulation. It is apparent from this relation that Ω^2 > 0 for all k > 0 whenPQ < 0. In this case Ω is real and waves are stable. However, when PQ > 0, Ω^2 < 0 then K^2 < (2Q/P)$|\phi_2|^2$ and waves are modulationally unstable. The maximum growth rate is obtained for $K = \sqrt{|Q/P||\Phi_0|}$ and is given by $\gamma_{max} = Im(\Omega)_{max} = Q|\phi_2|^2$. It is seen that instability sets in for perturbation wave length $\lambda > \lambda_c$, where $\lambda_c = {2\pi}/{K_c}$ and $K_c = \sqrt{|P/Q||\Phi_0|}$. Now we discuss the possible localized solitary wave solutions of (27). Since the wave packet can be stable or unstable in different conditions of θ , k , σ, β and α. P and Q can both be negative or they can have different signs. The latter condition implies two types of stationary solutions of NLSE. To obtain the profile in both cases, let us put

$$\phi = \rho(\zeta,\tau)e^{[\iota\sigma(\zeta,\tau)]} \quad (34)$$

where ρ and σ are two real variables. Substituting (34) into (27) and separating the real and imaginary parts and solve the resulting equation for ρ and σ. In case of modulationally unstable wave with P and Q having the same signs, we obtain the following envelope soliton solution

$$\phi(\xi,\tau) = \rho_m sech\left(\sqrt{\frac{1}{2}}\left|\frac{Q}{P}\right|\rho_m\zeta\right) \quad (35)$$

where ρ_m is constant and represents the nonlinear maximum amplitude. On the other hand with P and Q having the opposite signs, we have modulationally stable wave and obtain

$$\rho(\xi,\tau) = \rho_1\left[1 - b^2 sech^2\left(\sqrt{\frac{\rho_1}{2}}\left|\frac{Q}{P}\right|b\zeta\right)\right]^{1/2} \quad (36)$$

where $1 \geq b^2 = \rho_1^2 - (\rho_m^2)\rho_1^2, \rho_1$is a constant. Equation (36) represents an envelope hole sometimes called a dark soliton. Such solution corresponds to the accumulation of density in a region where wave intensity is very low. The parameter b determines the depth of the modulation. Further, when b = 1, we have

$$\rho(\xi,\tau) = \rho_1 tanh\left[\sqrt{\frac{\rho_1}{2}}\left|\frac{Q}{P}\right|b\zeta\right] \quad (37)$$

which is known as envelope shock. Coefficients P , Q of dispersion and nonlinear terms respectively are the functions of θ, nonthermal electrons distribution parameter β, ratio of hot electrons to cold electrons density a and wave number k. Therefore, one expects that θ and β will affect the unstable characteristics. We have chosen the following parameters: $n_{c0} = 0.5cm^{-3}$ $n_{h0} = 2.5cm^{-3}$ $\beta =$ 0,0.4,0.9 These parameters are within the range of observations from Viking satellite in the dayside auroral zone [10].It is noteworthy that wave packet will be unstable at higher wave numbers (k > 0.6) and at higherT_c/T_h.

The critical value of k for the onset of instability is lowered with increase of relative temperature T_c/T_h.This feature obviously highlights the crucial role of nonthermal electrons distribution as a major contributing factor to cause the modulational instability. In summary, a NLSE has been successfully derived to described finite amplitude EAW in a plasma composed of cold and hot electrons and stationary ions. The existence region of modulational instability of EAW has been investigated and the condition for appearance of MI has been given, that is, PQ > 0 and k > 0.6. The present study gives a simplified picture to understand the nonlinear phenomenon for the modulated EAW in plasma containing nonthermal electrons.

REFERENCES

[1] H. Derfler, T.C. Simonen, Phys. fluids 12, 269 (1969)

[2] D. Henry, R.A. Treumann, J. Plasma Phys. 8, 311 (1972)

[3] S. Ikezawa, Y. Nakamura, J. Phys. Soc. Jap. 50, 962 (1981)

[4] R.L. Mace, M.A. Hellberg, J. Plasma Phys. 43, 239 (1990)

[5] W.D. Jones, A. Lee, S.M. Gleman, H.A. Doucet, Phys. Rev. Lett. 35, 1349 (1975) 6. R.E. Ergun et al., Geophys. Res. Lett. 25, 2041 (1998)

[6] G.T. Delory, R.E. Ergun, C.W. Carlson, L. Muschielti, C.C.Chaston, J.P. McFadden, R. Strangewey, Geo- phys. Res. Lett. 25, 2069 (1998)

[7] P. Pottelette, M. Malinger, A. Bahnse, L. Eliasson, K. Stasiewicz, R.E. Erlandson, G. Markland, Ann. Geophys. 6, 573 (1988)

[8] M. Temerin, K. Cerny, W. Lotko, F.S. Mozer, Phys. Rev. Lett. 48, 1175 (1982)

[9] R. Bostrom et al., Phys. Rev. Lett. 61, 82 (1982)

[10] S.P. Gary, R.L. Tokar, Phys. Fluids 28, 2439 (1985)

[11] M. Berthomier, R. Pottelette, M. Malingre, Y. Khotyainsev, Phys. Plasmas 7, 2987 (2000)

[12] R.L. Tokar, S.P. Gray, Geophys. Res. Lett. 11, 1180 (1984)

[13] S.V. Singh, G.S. Lakhina, Planet. Space Sci. 49, 107 (2001)