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# A Review on the Mathematical Models to Evaluate the Properties of Fiber Reinforced Soil

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*Abstract:* **Soil reinforcement is most popular technique among other ground improvement technique. Availability of different form of materials as option for reinforcement is also making this technique popular among the engineers. Fiber reinforcement is one of the new emerging soil reinforcement techniques. It is similar to the reinforcement provided by the plant of roots. Randomly distributed fibers provide interlocking and friction resistance to resist the movement of soil particles, which significantly increase the load carrying capacity. Now-a-days fiber reinforcement is used in the embankment, slope stabilization, pavement application. Different studies were conducted to understand the behaviour of fiber reinforced soil. Also some mathematical models are developed. Mathematical model is useful to understand the behaviour of material and it is helpful for designing purpose. In this paper brief review of the mathematical models developed by different researchers is presented.**

*Keywords:* **shear strength, soil reinforcement, load carrying capacity**

## 1. INTRODUCTION

Ease of construction, overall economy and availability of different form of reinforcement as a option is making soil reinforcement a fast growing ground improvement technique. . Initially metals were used for reinforcement purpose, but now materials prepared from the polymers have accelerate the utilization of soil reinforcement. Major advantages of polymer based materials are less degradation rate and overall cost. Sheets, bars etc. are the traditional form of reinforcement. But Geocell, fibers which are new form of reinforcement are changing the face of soil reinforcement. Fiber reinforcement is being used from very ancient time as reinforcement technique. But due to lack of understanding of the strength improvement mechanism of fiber reinforcement, it was not used as construction material. But after many studies on fiber reinforced soil to understand the mechanism of fiber reinforcement have shown its potential benefit. In fiber reinforcement technique fibers are randomly mixed with the soil. Absence of weak plane due to isotropy, which is generally parallel to the soil reinforcement interface in other techniques, is major advantage of fiber reinforcement [1]. Fiber reinforced soil is also known as 'Ply Soil' [2]. Different soil parameters like; density of soil, soil type, size and shape of fiber, confining pressure and different fiber parameters like; length of fiber, fiber content, types of fiber, fiber surface characteristics etc. influence the performance of fiber reinforcement. Mathematical models based on the study are developed on the basis of understanding of the effect of these parameters on the behaviour of fiber reinforced soil. This paper is review of the different mathematical models developed by researchers.

## 2. MECHANISM OF FIBER REINFORCED SOIL

Fibers used for reinforcement purpose can be natural or synthetic type. Jute, coir etc. are example for natural fiber and polypropylene, glass fibers are example of synthetic fiber. Fiber reinforcement is similar to the root reinforcement where roots of plant binds the soil particle and resist their movement [3]. Fiber under stress condition deforms and penetration of soil particles on the fiber surface takes place. Due to which interlock and friction resistance mobilize. These two factors are primarily responsible for resistance of movement of soil particles, which enhances the load carrying capacity of fiber reinforced soils [4].

## 3. STUDY ON THE PARAMETRIC EFFECT

To understand the behaviour of fiber reinforced soil different studies based upon model tests, case study and triaxial tests have done. Case studies on roads by Lindh and Erriksson [5], on embankment by Gregory [6], 1998, Santoni et al. [7], on landfill liners by Refai [8] has shown the beneficial utilization of fiber reinforcement in different applications. Through model test on strip footing supported by fiber reinforced soil, improvement in the bearing capacity and stiffness have reported by Al-refai, [9] and Wasti and Butun [10]. Gray and Al-refai [11], Maher and Gray [1], Al-refai [12], Michalowski and Zhao [13], Ranjan et al. [14], Michalowski and cermak [15] through triaxial test have shown that performance of fiber reinforcement depends upon the fiber properties like length of the fiber, fiber content, stiffness of fiber and surface properties of soil and soil properties like soil type, size of particles etc.

To develop the understanding of behaviour of fiber reinforcement influence of each parameter were considered.

Studies based upon the fiber parameters have shown that fiber content and length of fiber has significant impact on the fiber reinforced soil. Increase in the fiber content increases the interaction between fiber and soil particles, due to more available surface area [11, 12, 14]. Due to greater interaction movement of particles are resisted, which increases the load carrying capacity. Similarly increase in the length of fiber also increases the interaction of soil particles and fiber. So, longer fibers can bind soil particle with greater resistance [12, 17, 18]. Surface properties and stiffness properties of fibers are also contribute in the reinforcement process of fiber. Fibers with rough surface have greater capability of strength improvement than smoother fibers [18, 19]. It is also found that load carrying capacity of fiber reinforced soil with stiffer fiber is greater [20]. Natural fiber and synthetic fibers both can be used for reinforcement purpose. Advantage with natural fiber is local availability and environmental friendly, while degradation with time is major disadvantage. Properties of natural fibers can changed with moisture content [21]. Synthetic fibers are relatively less biodegradable and their property doesn't change even in aggressive environment.

Mechanism of fiber reinforcement is based upon fiber-soil interaction, so soil properties also influence the properties of fiber reinforced soil. Different types of soil have different interaction behaviour. It is because of different size and shape of particles [19]. Sand has better interaction properties than the clay. Fine sands have greater contact area with fiber, so its interaction is higher. Shape of the soil particles of effects the performance of fiber reinforcement. Friction resistance is greater for the particles of angular shape than rounded soil particles. It is found that with increase in the angularity surface interaction increases [12]. With angularity critical confining pressure decreases or with increase in sphericity index critical confining pressure increases [1]. Due to angularity of soil particles interlock resistance increases. Gradation of soil indicates the range of particles present in the soil. If any soil is poorly graded then it has particles of same size. It is found that the well graded soil have better load carrying capacity than poorly graded soil. Since well graded soil have smaller to larger size of particles, interaction of soil and fiber will be greater. But surface interaction for poorly graded soil has lesser surface interaction [1]. Placement density influences the properties of fiber reinforced soil. Soil having greater density has better load carrying capacity [20]. Other than all these properties confining pressure have also impact on the strength behaviour fiber reinforced soil. With increase in the confining pressure, strength of fiber reinforcement increases. But it is also found that improvement in the peak stress with respect to unreinforced soil decreases with increase in the confining pressure. It shows that increased confining pressure suppress the reinforcing action of fiber [17].

## 4. MATHEMATICAL MODELS

Studies based upon fiber reinforced soils have shown the complex behaviour of fiber reinforcement. It depends upon different not only on fiber and soil parameters, it also depends upon the test conditions. Based upon the understanding and results of the tests different mathematical models based upon force equilibrium concept, energy concepts etc. are proposed. Many models are developed by the techniques of statistics. Some popular and useful models are presented in following sections.

**Waldron [3]** have proposed a model based upon force equilibrium to describe the reinforcing action of plant roots. This was used by **Gray and Ohashi [20]** explain the reinforcing of fiber reinforced soil. Gray and Ohashi [20] have conducted direct shear test on sand reinforced with different types of fibers to evaluate the strength improvement potential of fiber reinforcement. Parameters considered in this study are, fiber content, fiber area ratio and stiffness of fibers. To predict the shear strength of fiber reinforced soil a model based upon limit equilibrium of forces was proposed in this study. It is considered that during shearing process deformation process tensile stress in the fiber mobilized due to interaction between fiber and soil (Fig. 1). The mobilized tensile strength has two components one is tangential component and other is normal component. Tangential component directly resist the deformation and normal component increase the confining pressure which increase the shear strength of soil. Due to these two components shear strength of soil increases. Increase in the shear strength of soil can be expressed as:

For perpendicular fiber,

$$\Delta S_r = \sigma_t'[sin\theta + cos\theta . tan\emptyset]$$

For inclined fiber,

$$\Delta S_r = \sigma_t'[\sin(90 - \Psi) + \cos(90 - \Psi)\, tan\Phi]$$

$$\Psi = \tan^{-1}[\frac{1}{k + \tan i^{-1}}]$$

Where, $\Delta S_r$ = Shear strength increment from fiber reinforcement, $\sigma_t$' = mobilized tensile strength of fibers/ unit area of soil in shear, Φ = angle of friction, Θ = angle of shear distortion, k = (x/z) = shear distortion ratio, i = initial orientation of fiber with respect to shear surface, z = thickness of shear zone, x = horizontal shear displacement, $\sigma_t = (A_f/A)\sigma_t$, $A_f$ = area of fibers in shear, A = total area of soil in shear, $\sigma_t$ = tensile stress developed in fibers at shear plane.

Tensile stress developed in fiber can have parabolic or linear distribution. Stress distribution in both case can be expressed as follow:

For linear distribution $\sigma_t = [\frac{4E_f\tau' z(\sec\emptyset - 1)}{d}]^{1/2}$

For parabolic distribution $\sigma_t = [\frac{8E_f\tau' z(\sec\emptyset - 1)}{3d}]^{1/2}$

Here, z = thickness of shear zone, $E_f$ = modulus of fiber, d = diameter of fiber

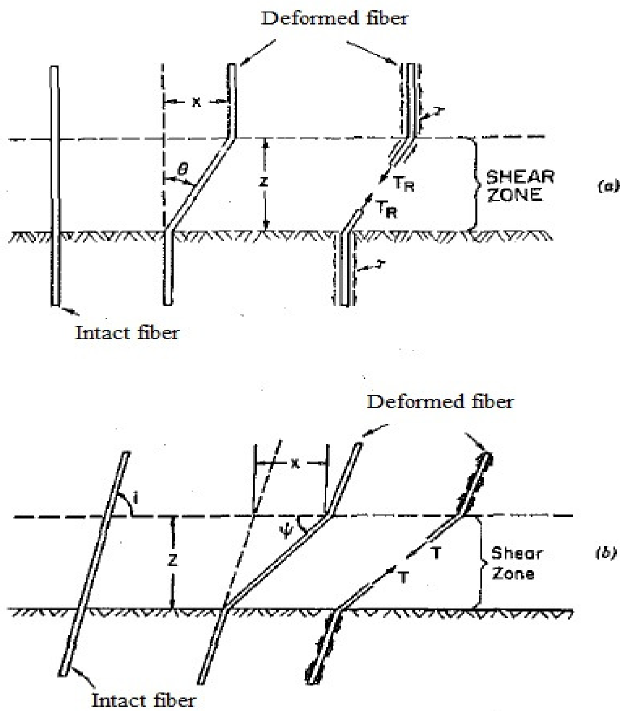


**Fig. 1 Deformation of fiber during shearing [1]**

Gray and Ohashi [20] have shown that the predicted value obtained from model and experimental results have good match. But there are limitations of the models like it doesn't include the concept of critical confining pressure, below which failure in the fiber takes place due to pullout of fiber. This model shows that the increase in the shear strength of soil is proportional area ratio of fiber or amount of fiber. But it is found from the results shown by Ranjan et. al. [14], Al-refai [12] and other researchers that after a certain amount of fiber strength improvement is not significant. This model has also not considered the other limitation related to length of fiber, confining pressure of soil and also not considered the soil related parameters.

**Maher and Gray [1]** have performed triaxial test on fiber reinforced granular soil. Effect of fiber properties and soil properties are evaluated in this study. In this study a mathematical model similar to the model proposed by the Waldren [3] is proposed. Expression for the mathematical model can be expressed as follows:

For $0 < \sigma_{conf} < \sigma_{critc}$ then,

$$\Delta S_R = N_s\left(\frac{\pi}{4}d^2\right)[(2\sigma_{conf}tan\delta)(\sin\omega + \cos\omega\tan\emptyset]\epsilon$$

For $\sigma_{conf} > \sigma_{crit}$

$$\Delta S_R = N_s\left(\frac{\pi}{4}d^2\right)[(2\sigma_{crit}tan\delta)(\sin\omega + \cos\omega\tan\emptyset]\epsilon$$

Here, $\sigma_{conf}$ = confining stress in triaxial chamber, $\sigma_{crit}$ = critical confining pressure, ΔS = increase in shear strength, $N_s$ = expected No. of fibers intersecting a unit area, ε = an empirical coefficient depending on sand granulometry (avg. Size $D_{50}$, particle sphericity ($S_p$), coefficient of uniformity ($C_u$) and fiber parameters; such as aspect ratio and skin, L = length of fiber, d = fiber diameter, δ = fiber skin friction, ω = angle of shear distortion ( $\tan^{-1}$ x/g), x = shear displacement parallel to shear zone, z = thickness of shear zone

No. of fiber crossing per unit area ($N_s$) is predicted on the basis of probabilistic approach proposed by Naaman [22]. $N_s = 2V_f/\Pi d^2$, $V_f$ = volume ratio of fiber.

This theory is suitable for the prediction of the shear strength improvement of fiber reinforced soil sample. This theory has taken care of soil parameters and fiber parameters both through empirical coefficient. It has also taken care of the critical confining pressure. But it is not explaining the optimum fiber content or optimum fiber length after which shear strength doesn't improve significantly.

**Ranjan et al [14]** have performed triaxial test on the fiber reinforced granular soil and evaluated the influence of different soil and fiber properties on the strength behaviour of granular soil. Through regression analysis based upon the results of the triaxial test relationship between the shear strength of the soil and different parameters are established. The expression obtained from the analysis can be expressed as follows:

When $\sigma_3 \leq \sigma_{crit}$ then,

$\sigma_{1f} = 12.3\,(w_f)^{0.4}(\frac{l}{d})^{0.28}(f^*)^{0.27}(f)^{1.1}(\sigma_3)^{0.68}R^2$ here, $R^2 = 0.903$

If $\sigma_3 \geq \sigma_{crit}$ then,

$\sigma_{1f} = 8.78\,(w_f)^{0.35}(\frac{l}{d})^{0.26}(f^*)^{0.06}(f)^{0.84}(\sigma_3)^{0.73}R^2$ here, $R^2 = 0.93$

Here, $f^* = (c_a/\sigma_n + \tan\delta)$ and $f = (c_a/\sigma_n + \tan\Phi)$

l = length of fiber, d = diameter of fiber, $\sigma_{1f}$ = shear strength of fiber reinforced soil, $w_f$ = wt. Fraction of fiber (%), $\sigma_3$ = confining pressure, Φ = angle of friction, δ = angle of friction.

This model predicts strength well when parameters are similar to the parameters used in this study. This model takes care of critical confining pressure, suppression of reinforcing action after optimum value of fiber length and fiber content.

**Zornberg [23]** has studied the behaviour of randomly distributed fiber reinforced soil. Mathematical model proposed in this study is based upon the discrete approach. In which it is considered that the load carrying capacity of fiber-soil mixture is due to the contribution by soil and through mobilized tensile stress of fiber. To predict the shear strength parameters of reinforced soil following expression was proposed:

Equivalent cohesion $c_{eq} = c[1 + \alpha X \eta c_{ic}]$

Equivalent friction $(\tan\phi)_{eq} = \tan\phi[1 + \alpha X \eta c_{i\phi}]$

$$X = \frac{V_f}{V} = \frac{X_w \gamma_d}{(1 + X_w) G_f \gamma_w}$$

Here, α = empirical coefficient to take into account the effect of fiber orientation, X = volumetric fiber content, η = aspect ratio of individual fibers and $X_w = f_w/w_s$ = gravimetric fiber content, $V_f$ = volume of fibers, V = total volume of fiber-reinforced soil, $G_f$ = specific gravity of fiber, $f_w$ = weight of fiber, $w_s$ = dry weight of soil, $\Upsilon_d$ = dry unit weight of fiber-reinforced soil, $\Upsilon_w$ = unit weight of water, $c_{ic}$ and $c_{i\Phi}$ = interaction coefficient for cohesion and friction coefficient.

This model directly gives the shear strength parameters of reinforced soil. It has taken care of fiber properties and soil properties related to interaction. It is helpful also to estimate the cohesive soil reinforced with fiber. But in this effect of critical confining pressure and effect of optimum fiber content and length is not considered.

**Michalowski and Cermak [15]** have conducted drained triaxial test on fiber reinforced sand. Model proposed in this study is based upon energy-based homogenization technique. In this technique work rate of the macroscopic stress is equated to the work dissipation rate in deformation process. Equivalent internal friction proposed in this study is as follow:

$$\phi = 2\tan^{-1}\left(\sqrt{\frac{\rho\eta\tan\phi_w + \sigma k_p}{\sigma - \rho\eta M\tan\phi_w}}\right) - \frac{\pi}{2}$$

Where $M = k_p \sin\Phi_o$, $k_p = \tan^2(45+\Phi/2)$, $\Phi_o = \tan^{-1}(k_p/2)^{-1}$, $\Phi_w$ = sand/fiber interface friction angle and ρ = volumetric fiber content.

**Babu and Vasudevan [17]** have conducted triaxial test on the tropical soil reinforced with coir fiber which is one of the natural fiber. Through non linear regression analysis on the results of triaxial test a mathematical model is established to predict the major principal stress at failure, cohesion, angle of friction, and initial stiffness of fiber reinforced soil. The expression is presented below:

$$\sigma_1 = [159.1 + 3.96\sigma_3 - 0.0083\sigma_3^2 - 2959d + 4866.5d^2 + 37.01w + 17.35w^2 + 58.8l - 1.69l^2 + 2.69\sigma_3 d + 548.61dw + 6.21wl - 0.016l\sigma_3]$$

$$c = 76.5 + 156.4d - 102.1d^2 + 126.1w - 39.3w^2 + 20.2dw$$

$$\emptyset = 23.1 - 78.55d + 191.1d^2 + 7.03w + 2.38w^2 - 15.02dw$$

$$E_i = [8992.2 + 64.94\sigma_3 - 0.14\sigma_3^2 - 94612d + 186594d^2 - 1744.9w + 1167.8w^2 + 1765.1l - 52.1l^2 + 33.3\sigma_3 d + 11707.7dw + 129.77wl - 0.47l\sigma_3]$$

Here, $\sigma_{1f}$ =major principal stress (kPa), $\sigma_3$ = confining pressure (kPa), d = diameter of fiber (mm), l = length of fibers (mm), w = fiber content, c = cohesion (kPa), Φ = angle of friction (degree), $E_i$ = initial stiffness modulus of fiber reinforced soil (kPa)

In this model results of synthetic and natural fiber were used in regression analysis. This model also provides the relation for stiffness, which is not established by previous studies. But it doesn’t take care of the critical confining pressure. Model is based upon the 56 test results, so relation is relatively conservative.

## 5. CONCLUSION

In this paper briefly discussion on the different models for the prediction of strength of fiber reinforced soil is presented. To get the better model it is important to understand the influence of different parameters on the behaviour of fiber reinforced soil. All models have some limitation, so during the utilization of particular model one should also understand the limitation of the particular model. Yet further research is required to understand the behaviour and for proper design of fiber reinforced soil structure and further improvement in the models are required.

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