Effect of Roughness Coefficient on Solution of Saint-Venant Equations in River Management

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ABSTRACT

The physical laws which govern two basic principles in the hydraulics of flow of water are principle of conservation of mass and principle of conservation of momentum. These two laws are of mathematical form generally expressed in partial differential equation form known as Saint-Venant equations. The solution of these equations are much complex. Conversion of these equations into ordinary partial differential equation forms and the simple discretization of this equation by implicit scheme using CFD tool are presented in this paper. During the time of flood, the flow in open channel is generally unsteady. Based on a simple implicit scheme using HECRAS computer model, the routing of flood in different section of study area in downstream locality is explored keeping the upstream flow hydrograph is as initial boundary condition. Upstream hydrograph at upstream boundary and the normal depth from manning equation at downstream boundary assuming uniform flow condition have been taken for the present analysis. Different stage hydrograph at downstream sections of the channel under sub critical flow conditions for different roughness coefficient are explored and discussed. The study will be helpful for obtaining the nature of stage and flow hydrograph of a channel for different roughness conditions.

Keywords: Momentum Equation, Continuity equation, implicit finite difference scheme, downstream condition, partial difference equation.

1. INTRODUCTION

Hydraulic flood routing problem is solved by saint-Venant equation which includes 1D unsteady continuity equation and momentum equation. These equations may be described by the complete equation of motion for unsteady non uniform flow, known as dynamic wave equation which is proposed by Saint-Venant in 1871[1]. The flow characteristics expressed by the momentum equation terms, are dimensionless (woolhiser&liggett, 1967)[2].In flood routing problems the Saint-Venant equation is solved by preissmann four point implicit finite –difference scheme in channel and flood plains(Rashid&chaudhry, 1995)[3]. Method of characteristics is applied for solving the 1-D shallow water equation which mostly used in explicit method (H.Eihanty, G.J.M

Copeland 2003)[4]. The principal objective of this report is to present descriptive data to choose manning's n in different conditions and its effect on solution of Saint-Venant equation in flood routing.

The unsteady flow needs to be solved by its magnitude and characteristics in channel and over bank area to predict the flow stage and discharge which requires an evaluation of roughness as it depends on bed material, cross sectional geometry, variation of channel cross section, obstruction like hydraulic structures, type and density of vegetation and degree of channel meandering etc. The factors which retard the flow along the channel causing the energy loss, increase the roughness coefficient while the factors accelerating the flow causing smoother flow condition, decrease the roughness coefficient. The determination of roughness coefficient in terms of Manning's n of a channel and its flood plains needs very much field experience. For selection of n value the basic knowledge of factors affecting roughness should also be known.

2. FLOOD ROUTING IN A RIVER SECTION – A CASE STUDY

For flood routing in a river section using numerical approaches need boundary condition for its solution. A hypothetical flood routing discharge hydrograph of upstream boundary in a wide rectangular river has been considered [9], here and is given by : A rectangular river section is assumed with width of the river (B) = 120m, Average longitudinal Bed slope (S_o) considered are 0.00061, Manning's roughness co-efficient n for the bed surface may vary time to time. For the present analysis we have taken 8 special cases of the field conditions details of which are described in later part of this paper. The values of Manning's roughness co-efficient *n for the bed surface are* 0.025, 0.030, 0.035, 0.040, 0.045, 0.050, 0.055, 0.060. Let the Base flow (Q_b) in the river to be 100 m³/sec. The U/S discharge hydrograph (Q_t) is generally sinusoidal in nature and is a function of time (t), base flow (Q_b) and peak flow(Q_p) is given by :-

$$Q(t) = \frac{Q_p}{2} sin\left(\frac{\pi t}{t_p} - \frac{\pi}{2}\right) + \frac{Q_p}{2} + Q_b \; for \; t < \; t_b$$

$$Q(t) = \frac{Q_p}{2} cos\left(\pi \frac{t - t_p}{t_b - t_p}\right) + \frac{Q_p}{2} + Q_b \; for \; t_p < t \leq \; t_b$$

$$Q(t) = Q_b \; for \; t > t_p \; . \qquad (1)$$

Where peak time (t_p), base time (t_b), and peak flow (Q_p) are assumed to be 5 hr, 15 hr, and 200 m³/sec respectively. Friction slope S_f is calculated from manning's equation

$$S_f = \frac{n^2 V^2}{R^{4/3}}, \tag{2}$$

Where n=manning's roughness coefficient, R=A/P is the hydraulic radius; A is the flow area; P is the wetted perimeter.

3. GOVERNING EQUATIONS

The Saint-Venant equations (both continuity equation and momentum equations) for one dimensional steady and unsteady flow of river are given by:

$$\frac{\partial y}{\partial t} + D_h \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0 \tag{3}$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} = g(S_o - S_f) \tag{4}$$

Where V is the flow velocity; y is the flow depth;

$D_h = A/B$ is hydraulic depth; A is flow area, B is top width of the channel ; S_o is the channel bottom slope, S_f is the slope of energy grade line; x is the distance along the channel length; t is time and g is the acceleration due to gravity.

These equations can be solved by the River Analysis System (HEC-RAS) software which use the Preissmann implicit scheme. The model and methodology of the schemes are described below.

4. MODEL DEVELOPMENT AND METHODOLOGY

The most successful and accepted procedure for solving the one dimensional unsteady flow equation is the four point implicit scheme also known as the box scheme . Under the scheme, methods of solution are as follows

$$f_i = f_i^j \tag{5}$$

$$\Delta f_i^j = f_{i+1}^{j+1} - f_i^j \tag{6}$$

Or

$$f_i^{j+1} = f_i^j + \Delta f_i^j \tag{7}$$

size of grid taken as (i, j) ; where
i=space interval, j= time interval

General implicit finite difference scheme forms are

1.Time derivative

$$\frac{\partial f}{\partial t} \approx \frac{\Delta f}{\Delta t} = \frac{0.5(\Delta f_{i+1} + \Delta f_i)}{\Delta t} \tag{8}$$

2. Space derivative

$$\frac{\partial f}{\partial x} \approx \frac{\Delta f}{\Delta x} = \frac{(f_{i+1} - f_i) + \theta(\Delta f_{i+1} - \Delta f_i)}{\Delta x} \tag{9}$$

3. Function value

$$f = 0.5(f_{i+1} + f_i) + 0.5\theta(\Delta f_{i+1} + \Delta f_i) \tag{10}$$

f refers to both V and y in the partial derivatives and f stands for S_f and V as a coefficient .

A. Continuity Equation

$$\frac{\partial A}{\partial t} + \frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} - q_l = 0 \tag{11}$$

A=cross sectional area, x=distance along the channel; Q=flow, t=time, S=storage from non-conveying portions of cross section, q_l= lateral inflow per unit distance

The above equation can be written for channel or flood plain :

$$\frac{\partial Q_c}{\partial x_c} + \frac{\partial A_c}{\partial t} = q_f \tag{12}$$

$$\frac{\partial Q_f}{\partial x_f} + \frac{\partial A_f}{\partial t} + \frac{\partial S}{\partial t} = q_{c+} q_l \tag{13}$$

Where the subscripts c and f refer to the channel and flood plain, respectively

q_l is the lateral inflow per unit length of flood plain and $q_c\ and\ q_f$ are the exchange of water between the channel and flood plane.

B. Momentum Equation

The equation states that the rate of change in momentum is equal to the external forces acting on the system. For single channel

$$\frac{\partial Q}{\partial t} + \frac{\partial (VQ)}{\partial x} + gA\left(\frac{\partial Z}{\partial x} + S_f\right) = q_{c\,+} q_l \tag{14}$$

Where g= acceleration due to gravity, S_f =friction slope, V= Velocity

The above equation can be written for the channel and for the flood plain:

$$\frac{\partial Q}{\partial t}+\frac{\partial (VQ)}{\partial x}+gA\left(\frac{\partial Z}{\partial x}+S_f\right)=q_c+q_l \qquad (15)$$

5. SELECTION AND APPLICATION OF ROUGHNESS COEFFICIENT

From the information which has been accumulated by the U.S geological survey are available by all hydraulic computations involving flow in open channel. They have computed the roughness coefficient for different channels .The selection of roughness co-efficient are thus useful in estimating the roughness characteristics of similar channel .So it will be very helpful to engineers to consider and apply a roughness coefficient where the geometry, appearance and roughness characteristics of the channel are known .

Sites are arranged according to the value of the computed roughness coefficient, in ascending order

Manning *n* =0.024-0.027 Bed consists of slime- covered cobbles and gravel deposits over smooth to rough rock, banks are composed of clay, short grass and exposed tree roots.

Manning *n* =0.028-0.032 Bed consists of well-rounded boulders sand, gravel, and boulders ; d_{50} = 135-175mm, d_{84} = 205- 325 mm. Banks are composed of gravel and boulders, and have tree and brush cover.

Manning *n* =0.033-0.036 Bed consists of coarse gravel and cobbles with scattered boulders with some exposed bedrock. Channel is bordered by railroad on the right and highway on the left. Banks are gravel and rock and have light vegetation cover.

Manning *n* =0.036-0.041 Bed is gravel and boulders well-rounded small boulders. d_{50} = 172-195 mm, d_{84} = 265-360 mm . Banks are lined with overhanging bushes, trees.

Manning *n* =0.041-0.043 Bed is composed of sand, gravel, and small boulders with scattered large angular rocks d_{50} = 93 mm- 142 mm, d_{84} =157 mm- 285 mm. Banks are composed of gravel and boulders, and have trees and brush along the tops. Banks are irregular and eroded, and have sparse cover of grass .

Manning *n* =0.044-0.050 Bed is sand and gravel with several outcrops in the reach and mostly coarse sand. Banks are steep and lined with overhanging trees and bushes angular boulders as much as 2 ft in diameter and fairly steep and contain medium growths of underbrush and large trees.

Manning *n* =0.051-0.060 Bed is mostly rock and very irregular size of 5 ft in diameter of coarse sand and a few outcrops . Banks are lined with boulder, small trees, and bushes and are heavily lined with overhanging birch trees. Bed and bank consist of boulders; d_{50} = 210 mm, d_{84} = 375 mm.

6. RESULT AND DISCUSSION

The initial condition for the model corresponds to uniform flow with discharge 99.59 m³/s and flow depth 0.86 m. Also friction slope is computed using manning's equation with varying roughness coefficient. The upstream discharge hydrograph is used as boundary condition for the model.

Discharge and stage hydrograph at the 16 kms from upstream end for different manning's n obtained from HEC-RAS model are shown in Fig.1-Fig.4.the initial flow for the all manning's n corresponds to uniform flow with discharge 100 m³/sec and stage 0.86 m .Also the friction slope is computed using manning's equation with different roughness coefficient which is used at downstream boundary condition. Fig. 1 and Fig. 2 show the flow values for different manning's *n* 0.025 to 0.055 and 0.030 to 0.060 respectively, where the flows are decreased for higher n values for a certain period causes the reduction of peak flow. But after that the flows are increasing to discharge the same volume of water from river. Measured velocities are decreased because of the obstruction in river bed and roughness in bank causing the reduction in velocity. To discharge the same volume of water through the channel the flow depth is rises accordingly. So the higher roughness lowers the velocity of flow.

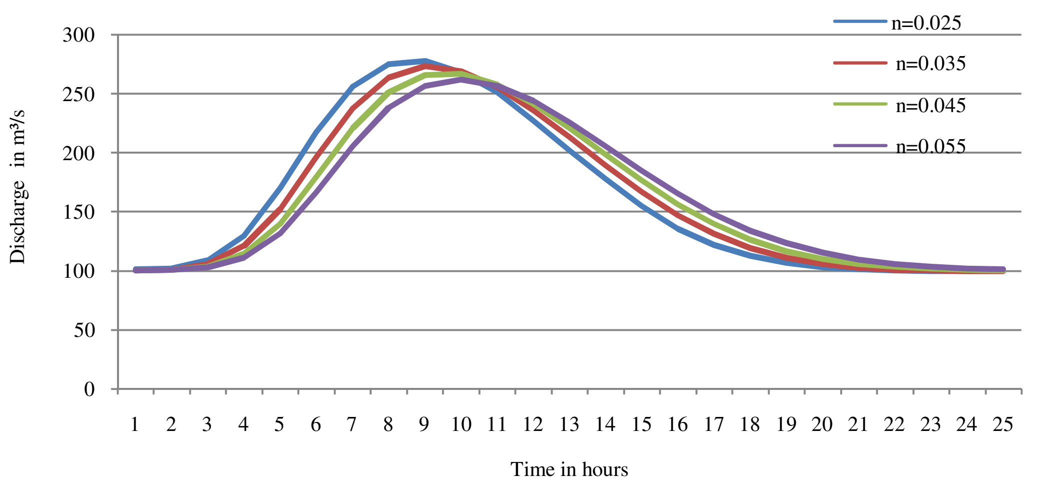

Fig. 1 Measured flow hydrograph at 16 kms section for different manning's *n* using HEC-RAS model

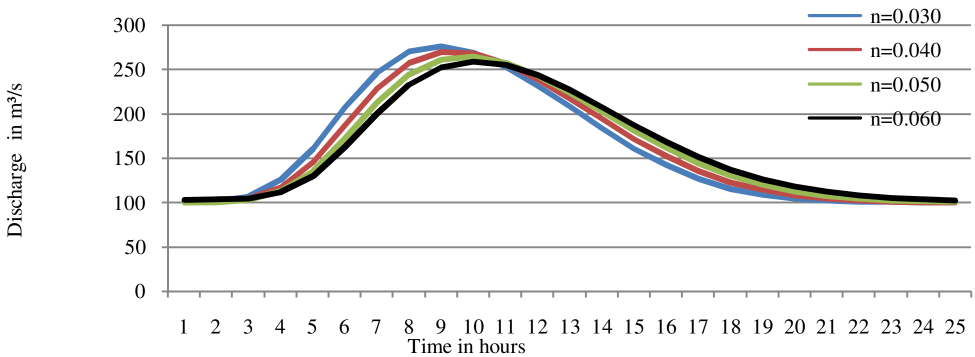

Fig. 2 Measured flow hydrograph at 16 kms section for different manning's *n* using HEC-RAS model

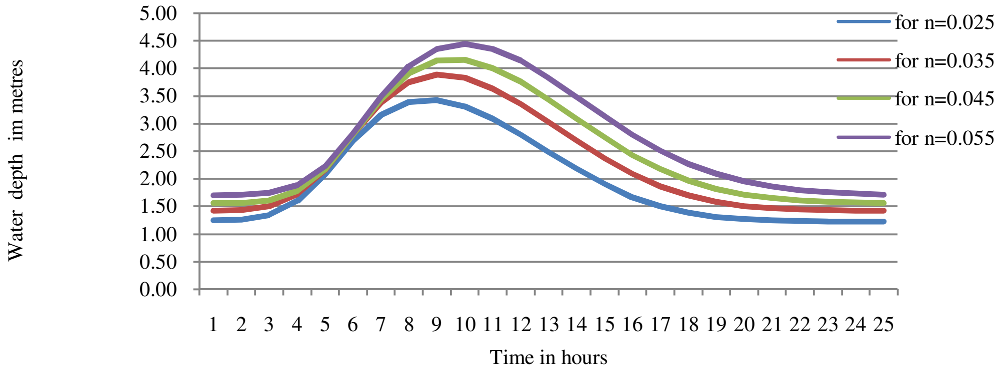

Fig. 3 Measured stage hydrograph at 16 kms section for different manning's *n* using HEC-RAS model

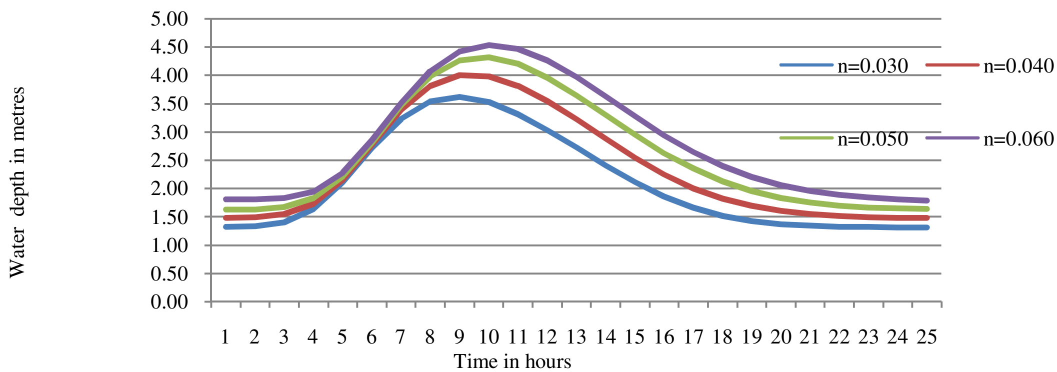

Fig. 4 Measured stage hydrograph at 16 kms section for different manning's *n* using HEC-RAS model

The graphs obtained for stage at a section 16 kms from upstream end for different *n* value are shown in Fig. 3 and Fig. 4. It is shown that the water depth is increased due reduction of velocity for higher *n* values to discharge the same amount of water along the channel. Discharge & stage hydrograph at the 16 kms section for *n* equal to 0.025to 0.060 are shown in Fig. 1 to Fig. 4.The discharge and depth values obtained from HEC-RAS model are given in TABLE 1. It can be seen that the measured peak flow discharge is lower with increase of manning's *n* values at a section distance 16 kms from upstream end..It can be seen that the stages are gradually increased with increase in values. It is also observed that the peak flow depth measured is more with increase in manning's *n*.

TABLE 1 : Flow values (Q) and water depth (y) for different manning's *n*

Time (hrs)	'Q' in [m³/s] and water depth (y) in [m]for different manning's *n*															
	n=0.025		*n*=0.030		*n*=0.035		*n*=0.040		*n*=0.045		*n*=0.050		*n*=0.055		*n*=0.060	
	Q	y	Q	y	Q	y	Q	y	Q	y	Q	y	Q	y	Q	y
1	101	1.25	100	1.31	100	1.42	100	1.49	100	1.56	100	1.63	100	1.70	103	1.80
2	102	1.26	101	1.32	101	1.43	101	1.49	101	1.56	100	1.63	101	1.71	103	1.81
3	109	1.34	106	1.39	106	1.50	104	1.55	103	1.61	103	1.68	103	1.74	104	1.83
4	130	1.60	125	1.64	121	1.72	117	1.73	114	1.78	113	1.84	111	1.88	111	1.94
5	170	2.10	160	2.10	152	2.16	145	2.15	140	2.17	135	2.21	132	2.23	130	2.27
6	218	2.68	207	2.71	196	2.79	187	2.77	180	2.79	172	2.81	166	2.82	162	2.85
7	256	3.16	247	3.23	238	3.38	229	3.39	221	3.43	213	3.47	206	3.49	200	3.51
8	275	3.39	270	3.54	263	3.75	257	3.81	251	3.90	244	3.99	238	4.03	233	4.08
9	278	3.43	276	3.62	273	3.88	270	4.00	266	4.13	261	4.26	256	4.35	252	4.42
10	268	3.31	269	3.53	269	3.82	268	3.98	267	4.15	265	4.32	262	4.44	259	4.53
11	251	3.10	253	3.31	255	3.63	257	3.81	257	4.00	258	4.20	256	4.35	255	4.47
12	227	2.81	232	3.03	236	3.35	239	3.54	241	3.75	243	3.96	244	4.14	243	4.26
13	202	2.49	208	2.72	213	3.03	217	3.22	221	3.43	223	3.65	226	3.83	227	3.97
14	178	2.19	184	2.41	189	2.69	194	2.88	198	3.08	202	3.30	205	3.48	207	3.63
15	155	1.91	161	2.11	167	2.37	172	2.54	177	2.75	181	2.95	185	3.13	187	3.28
16	136	1.67	142	1.85	147	2.09	152	2.25	156	2.43	161	2.63	165	2.80	168	2.94
17	122	1.50	126	1.65	131	1.86	135	2.00	140	2.17	144	2.35	148	2.51	151	2.65
18	113	1.39	115	1.51	119	1.69	123	1.82	126	1.96	130	2.12	134	2.27	137	2.40

19	106	1.31	108	1.42	111	1.58	114	1.69	116	1.81	120	1.96	123	2.09	126	2.20
20	103	1.27	104	1.37	106	1.50	108	1.60	110	1.71	112	1.83	115	1.95	118	2.06
21	101	1.25	102	1.34	103	1.46	104	1.55	106	1.65	107	1.75	110	1.86	112	1.96
22	100	1.24	101	1.32	101	1.44	102	1.52	103	1.61	104	1.70	106	1.79	108	1.89
23	100	1.23	100	1.31	100	1.42	101	1.50	102	1.58	102	1.67	103	1.75	105	1.84
24	100	1.23	100	1.31	100	1.42	100	1.49	101	1.57	101	1.65	102	1.73	103	1.81
25	100	1.23	100	1.31	99.7	1.42	100	1.48	100	1.56	100	1.64	101	1.71	102	1.79

7. CONCLUSION

In this research, the solution of fully saint-venant equation in unsteady flow routing problems for different roughness in terms of Manning's *n* using HECRAS Numerical computer model are presented. From the analysis, it can be seen that there is a decrease in velocity with increase in *n* value. So the results of flow values are decreased and water level are increased with increase of *n* values the measured output of stages are increased when the flow values are decreased, for discharging the same amount of water at downstream locality. There is a possibility of spreading of flood in downstream area as the water stages are increased with n values.

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