Regulation of Unsteady Flow in Open Channel by using Inverse Explicit Method and Comparison with HECRAS

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ABSTRACT

Flood routing and operation-type problems are two major problems which are required to be solved frequently in unsteady flow problems in open channels. Now a days for routing and regulating problem, the Saint-Venant equations is used to predict the discharge and water stage at the study area in the channel. Routing of flood calculates the discharge and flow depth at future time series. On the other hand, the operation problem is mainly used to compute the inflow at required upstream section for the regulating structures of the delivery system to get a predefined water demand at required section at downstream end of the channel. So it is the inverse computational problem from downstream to the study section at upstream. So an explicit finite difference scheme which is solved from downstream to upstream also known as inverse explicit scheme is presented to solve the operation-type problems in open channels. The finite difference method inverse explicit scheme is applied to solve the Saint-Venant equations based on the discretization of the Preissmann scheme. The finite difference inverse explicit model is applied to a rectangular canal. The computation is performed by proceeding first backward in time and then backward in space from downstream. The method is numerically stable and is compared by the HEC-RAS commercial computer model.

Keywords: Operating problems, Saint-Venant equations, explicit finite difference method, inverse computational problem, Preissmann scheme, discretization, HECRAS

1. INTRODUCTION

Generally the flow in open channels is unsteady. In operational type problem, the main work is to regulate the flow in various problems such as navigation and operation of irrigation and power canals. Expression of the principles of conservation of mass and momentum are required for analysis of the unsteady flow which change its flow characteristic with time. By numerical method the unsteady flow is solved very accurately so that the results obtained are practically applied in any problem case. Mathematical models of unsteady flow in open channels applied the fully dynamic Saint Venant equation for analyzing. In irrigation system operational type problems are focused now a days. In hydraulics, control of water is becoming very important due to the increasing of water demand for every purpose. So for engineering purposes, the governing equations i.e. Saint Venant equations are used which are usually built Precise. Any control should be done in the upstream inflow so that losses of water or shortage of water could not be found at required demand area. The main objective of operation along irrigation canals aims for regulating structures at upstream to maintain a required water demand at required downstream. Mathematical model is used to predict the upstream inflow according to the required downstream flow precisely. The partial differential Saint Venant equations, which express the both principles of conservation of mass and momentum, can be solved at a finite number of grid points in the rectangular channel.

2. PROBLEM STATEMENT

The performance of the inverse explicit finite difference scheme is tested using an unsteady flow data in a rectangle channel with a bottom width of 5.0m. The bed slope of the channel is 0.001, roughness coefficient is assumed as (Manning's n) = 0.025, the channel length is 2.5 m, and a fixed overflow weir with free flow condition is considered as a downstream outlet. At the required downstream outlet, the flow increases from 5m^3/sec to 10 m^3/sec in one hour and it remains constant at 10 m^3 /sec for the next two hours, then decreases to 5.0 m^3/sec in one hour. Considering it as the demand at downstream outlet the upstream inflow rate is found out by the inverse explicit finite difference scheme. The water depth values at the downstream end section also are given. In the absent of depth values a discharge-water depth relationship can be used to obtain the water depth at the downstream end section. The flow and water stage at the upstream intake are to be computed using the specified discharge and water depth at the downstream end section as the boundary conditions. For the initial conditions the lateral inflow of 5m^3/sec are considered.

3. GOVERNING EQUATION

In open channel Unsteady flow commonly known as the Saint-Venant equations(1871). The Saint-Venant equations can be formulated in various ways, depending on the assumptions used in their derivations [1]. Assuming no lateral outflow, these equations can be written as:

$$\frac{\partial y}{\partial t} + \frac{1}{b}\frac{\partial Q}{\partial x} = 0 \qquad (1)$$

And the momentum equation is given by:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x} + S_f - S_0\right) = 0 \qquad (2)$$

where: A = wetted cross-sectional area; b = wetted top width; g = gravitational acceleration; Q = discharge (through A); y = depth of flow ; t = time; x = space; S_0 =bottom slope of the channel and S_f = friction slope.

Methodology

The conservation form of the governing equations [2], in the matrix form may be written as

$$X_t + Y_x + Z = 0 \quad (3)$$

in which

$$X = \begin{bmatrix} A \\ Q \end{bmatrix}; Y = \begin{bmatrix} Q \\ QV + gAy' \end{bmatrix}; Z = \begin{bmatrix} 0 \\ -gA(S_0 - S_f) \end{bmatrix}$$

and Ay'=moment of flow area about the free surface

By Inverse explicit finite difference scheme the operation problems can be solved. For the operation problems expected discharge and water level at downstream boundary condition are required. The inverse explicit scheme is an explicit solution based on preissmann scheme which is given by:

$$\frac{\partial f}{\partial t} = \emptyset \frac{f_{i,j-1} - f_{i-1,j-1}}{\Delta t} + (1-\emptyset)\frac{f_{i,j} - f_{i-1,j}}{\Delta t} \quad (4)$$

$$\frac{\partial f}{\partial x} = \theta \frac{f_{i-1,j} - f_{i-1,j-1}}{\Delta x} + (1-\theta)\frac{f_{i,j} - f_{i,j-1}}{\Delta x}$$

$$F(x,t) = \theta \left[f_{i-1,j-1} + f_{i-1,j}\right] + (1-\theta)\left[f_{i,j-1} + f_{i,j}\right]$$

Where $f_{i,j} = f(i\Delta x, j\Delta t)$. In which Δx = Space interval and Δt =time interval θ and ϕ is weighting coefficient; f refers to both V and y i.e. *Q and F stands for* S_f.

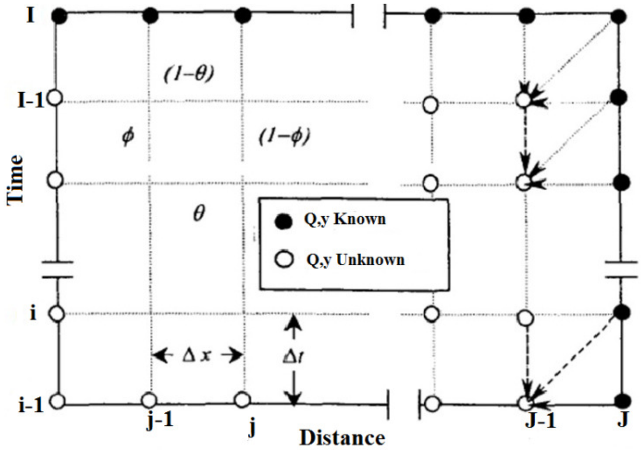

Fig 1. Computational grid of inverse explicit scheme

Considering the time level I as the final condition and knowing all discharges and the corresponding depths of downstream level are required. The discharge & water depth profile at the time level I-1 can be computed by proceeding first backward in time & then backward in space. Now introducing the preissmann scheme in (1) and (2) yields the following linear algebraic equations for each two adjacent grid points [4] [5].

$$P_1 q_{i-1} + Q_1 q_i + R_1 y_{i-1} + S_1 y_i + T_1 = 0 \quad (5)$$

$$P_2 q_{i-1} + Q_2 q_i + R_2 y_{i-1} + S_2 y_i + T_2 = 0 \quad (6)$$

Where q_i and y_i =discharge and water level increment from time level i to i-1 ; q_{i-1} and y_{i-1} are these at grid point i-1 ; and P, Q, R, S and T are coefficient computed with known values at time level 'i' . For operation problems, there are two downstream boundary conditions specified, the expected discharge and water depth at the downstream outlet. Knowing q_j and y_j between any two time levels at the last section of the channel, one can apply Eq. (5) and Eq.(6) for the last two sections *j*-1 and *j* (Fig. 2), and solve q_{j-1} and y_{j-1} explicitly:

$$q_{j-1} = \frac{R_1(Q_2 q_j + S_2 y_j + T_2) - R_2(Q_1 q_j + S_1 y_j + T_1)}{P_1 R_2 - R_1 P_2} \quad (7)$$

$$y_{j-1} = \frac{P_1(Q_2 q_j + S_2 y_j + T_2) - P_2(Q_1 q_j + S_1 y_j + T_1)}{R_1 P_2 - P_1 R_2} \quad (8)$$

With the calculated value of q_{j-1} and y_{j-1} the value of q_{j-2} and y_{j-2} can be computed [3]. This computation process is continued until the upstream boundary is reached, as shown in Fig. 1. The foregoing is a backward computation in space. This computing procedure cannot be applied easily because the changes of flow at the downstream end of the channel are caused by the changes upstream .If the time taken by the first propagation to travel from upstream to the downstream end of the channel is T, the discharge and water level at the downstream-end section remain the same before the time level (t_0+ T), and the process between t_0 and (t_0 + T) cannot be computed. Since the flow state at the time level (t_0 + T) is required for the computation after that time level, the problem cannot be solved. Fortunately, the aforementioned difficulty can be overcome by backward computation in time. Considering the time level (t_0 + jΔt) as the final condition, and knowing q_j and y_j between any two time levels at the downstream-end section, one may proceed backward in both space and time to compute the discharge and water-level profile at the time level (t_0 + (j−1)Δt)

This computation process is continued until time level t_0. The solution gives the discharge and water level in the channel at each section and defines the flow pattern at the upstream intake required to match the demand of the downstream flow. The physical meaning of the backward-computation method is very clear. Knowing the expected outflow at the downstream outlet, one has to look backward, in both space and time, for the necessary upstream inflow. With the Preissmann scheme in this form as mentioned above in Eq.4, the coefficients P, Q, R, S and T in Eq. 7 and Eq.8 are computed from the later time level instead of the earlier time level. The discharge and depth values obtained from inverse explicit method is compared with values obtained from HECRAS .

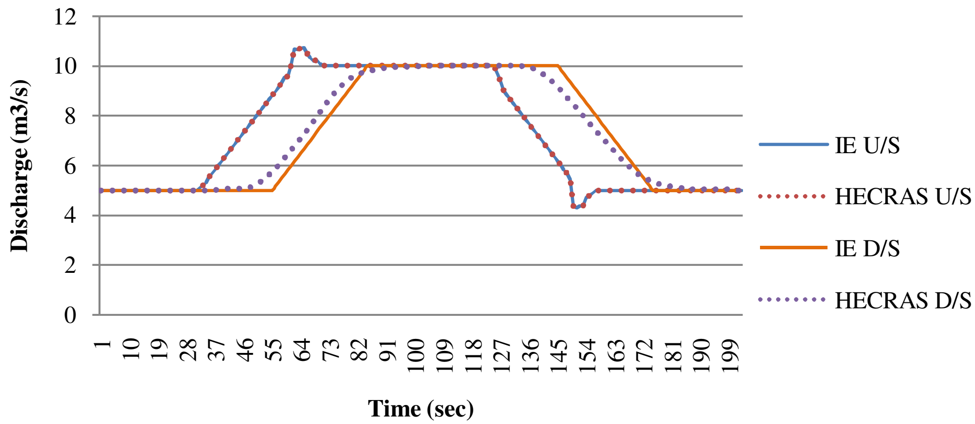

Fig. 2 Comparison between Discharge (Q) Hydrographs Using Inverse Explicit(IE) Scheme and HECRAS

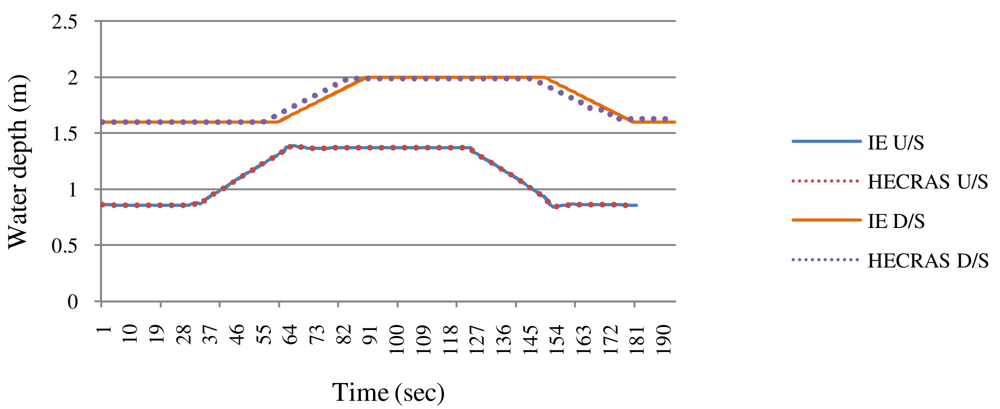

Fig.3 Comparison between Water Depth (y) Hydrographs Using Inverse Explicit (IE) Scheme and HECRAS

4. RESULT AND DISCUSSION

By satisfying the courant numbers for stability check the spatial grid size along the canal is taken as 100m and time grid size is 120 sec. Taking the given downstream demand hydrograph as the downstream boundary condition, the upstream hydrograph is determine by inverse explicit method. For comparison of the results, HECRAS software is used. In HECRAS software, the upstream hydrograph is required. So the upstream hydrograph obtained from IE method is used as the upstream hydrograph. Then the downstream hydrograph is determined. The flow and depth hydrograph are shown in Fig. 2 and Fig. 3 respectively. And the corresponding values of flow and depth are given in TABLE 1. The demand hydrograph at downstream is same as the downstream hydrograph obtained from HECRAS. The results coming from inverse explicit method are for the regulation and release of flow at upstream, so that the demand at observation point is fulfilled. It has been observed from the values shown in the TABLE.1 that there is a good match in present approach and the HEC-RAS computer model. But the results obtained from inverse explicit method are 20 minutes later than results obtained from HEC-RAS model. This is very negligible for applications point of view.

TABLE 1 : Values of Discharge (Q) in m^3/s and water depth (y) in metre at D/S and U/S for Inverse explicit scheme and HECRAS are given

Time (minutes)	At downstream end				At upstream end			
	Inverse explicit method		HEC-RAS		Inverse explicit method		HEC-RAS	
T	Q	y	Q	y	Q	y	Q	y
0	5.00	1.60	5.00	1.60	5.00	0.86	5.00	0.86
20	5.00	1.60	5.00	1.60	5.00	0.86	5.00	0.86
40	5.00	1.60	5.00	1.60	5.00	0.86	5.00	0.86
60	5.00	1.60	5.00	1.60	5.00	0.86	5.00	0.86
80	5.00	1.60	6.33	1.60	6.33	0.98	6.33	0.98
100	5.00	1.60	8.00	1.60	8.00	1.15	8.00	1.15
120	5.83	1.60	9.56	1.67	9.56	1.30	9.56	1.30
140	7.50	1.73	10.10	1.80	10.10	1.37	10.10	1.37
160	9.17	1.87	10.00	1.93	10.00	1.37	10.00	1.37
180	10.00	2.00	10.00	1.99	10.00	1.37	10.00	1.37
200	10.00	2.00	10.00	1.99	10.00	1.37	10.00	1.37
220	10.00	2.00	10.00	1.99	10.00	1.37	10.00	1.37
240	10.00	2.00	10.01	1.99	10.01	1.37	10.01	1.37
260	10.00	2.00	8.66	1.99	8.66	1.27	8.66	1.27
280	10.00	2.00	7.00	1.99	7.00	1.11	7.00	1.11
300	9.17	2.00	4.35	1.93	4.35	0.94	4.35	0.94
320	7.50	1.87	5.00	1.81	5.00	0.87	5.00	0.87
340	5.83	1.73	5.00	1.71	5.00	0.86	5.00	0.86
360	5.00	1.60	5.00	1.63	5.00	0.86	5.00	0.86

5. CONCLUSION

From the present research it has been concluded that the application of inverse explicit method in regulation of unsteady flow in a canal is suitable. As the peak flow and peak flow depth values are matching in both in present approach i.e. inverse explicit scheme and HECRAS, so it is concluded as user friendly. It can be applicable for any other canal or small channel having their gates for regulation.

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