A Finite-Element Method of Analysis for Composite Beams

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ABSTRACT

A method of analysis for composite beams with any degree of horizontal shear interaction is presented. The method is applicable to composite beams that have sudden, point by point variations in their structural properties. Also, the beam may be subjected to any type of load whether transverse or longitudinal and may be assumed to be supported in any reasonable manner. The paper includes a finite element model substituted for the real structure and the mathematical matrix equations which describe the load –deflection behavior for the model. These equations are solved for the unknown deflections by modified Gaussian elimination method.

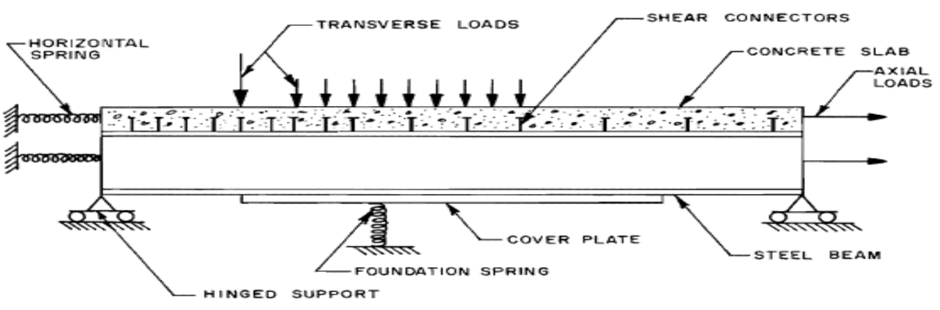
Keywords. Finite element model, bar and spring model, gauss elimination method, composite beams

1. INTRODUCTION

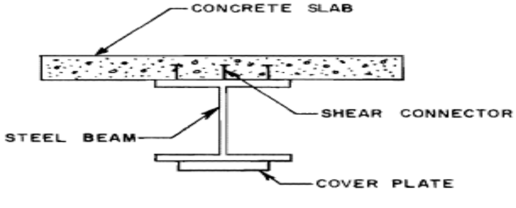
This study is concerned with the development of an efficient method for the analysis of composite beams. In this text, the term "composite beam" refers to structural systems consisting of two separate members that are joined at their interface by a shear connection. A practical example is a highway bridge girder that acts compositely with the floor slab. A typical composite beam is shown in Fig 1. The top member is a concrete slab and the bottom member is a steel I-beam. Shear connection is provided between the two members by studs which were welded to the top of the beam prior to placement of the concrete. The method of analysis presented is not limited to concrete- steel combinations such as shown in Fig 1 but is applicable to any similar composite system. The stiffness and strength characteristics of a composite system are greatly affected by the amount of interaction between the slab and the beam. Number, location, and strength of the shear connectors are the factors that determine the degree of interaction between the two members. A complete absence of shear connectors causes the most flexible system. At the other extreme, the stiffest possible system is obtained when sufficient connectors are provided to insure that there is no slip between the two members. It is possible to determine the moment of inertia of the system for both of the extreme cases; therefore, conventional methods of analysis may be applied to them. For intermediate cases, it is not possible to calculate the moment of inertia of the system; hence, a new method of analysis is needed. Special-case solutions for partial-interaction problems may be found in the technical literature, but a general method of analysis for the full range of composite structures has not been found

2. A FINITE-ELEMENT METHOD OF ANALYSIS FOR BEAM-COLUMNS

The method of analysis presented in this text has been greatly influenced by Matlock's numerical solutions to beam-column on elastic foundation problems. Matlock's approach to these problems is to replace the real physical system by an appropriate finite-element model. The model used by Matlock is composed of rigid, weightless bars hinged at their ends. The beam stiffness of each finite beam element is concentrated in the springs at the hinges. In Fig 2 the development of a bar-and-spring model from a section of a beam element subjected to pure bending is shown. Figure 2b shows the stresses acting on the beam element. The distributed stresses may be replaced by concentrated forces as shown in Fig 2c. In Fig 2d the deformed beam element is replaced by a pair of plates hinged at the center and restrained by springs at the top and bottom. A beam could be represented by a series of such beam-element models as in Fig 2e. Finally, a cruder model could be made by using rigid bars and springs as shown in Fig 2f. Based on the model, a set of equations which describes the deflections as a function of the applied loads is derived. This set of equations forms a diagonally-banded matrix which is solved by a direct elimination procedure.



a. Elevation.



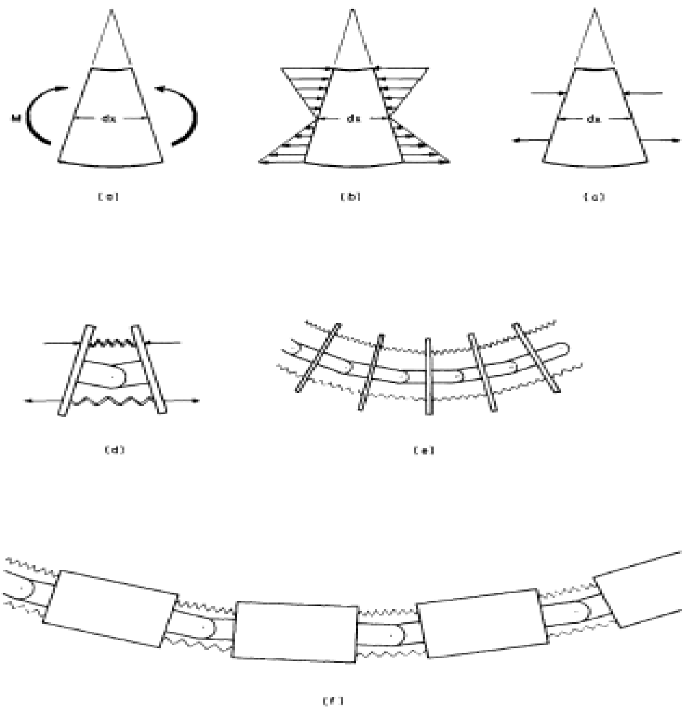


Figure 1 &2

A list of the assumptions that were made in the derivation of the equations are : vertical deflections of the slab and beam are equal, the slab and the beam interface is a straight line, deflections are small compared to the length of the structure, linearly-elastic shear connectors are used, slab and beam have linear stress-strain properties, the strain distribution throughout the cross section of both the beam and slab is linear; however, the strain distribution for the entire composite section may have a discontinuity at the interface as shown in Fig 3d, transverse shear deformations are negligible within each member, the cross sections of both members are symmetrical about the vertical axis and loads are applied only in the plane of the vertical axis.

3. THE BAR-AND-SPRING MODEL

Figure4 shows the model that is used to replace the real system. Each member (slab and beam) is represented by a system of bars and springs. All of the bending characteristics for each of the two layers of the system are lumped in the springs which act at the hinges of the bars. The weightless bars possess an infinite resistance to bending, but they are axially deformable. Pin-connected vertical spacer rods are included between the slab model and the beam model to insure that their vertical deflections are equal. The horizontal shear transfer mechanism is modeled by a pointer rod and spring system. To the center of each bar is attached an infinitely stiff cantilever pointer rod that extends to the interface. A linear spring which represents a shear connector is attached to each pair of slab and beam pointer rods. An important feature of the model shown is that it permits a completely general description of the system. The properties of the system are defined only at discrete points; some properties are related to the joints while others are related to the bars. Therefore, abrupt variations in the properties along the member are allowable.

The following quantities are defined at the joints: vertical deflection W, bending moments M^S and M^b, accumulated axial transverse loads Q, applied torques T^S and T^b, rotational restraints R^S, and R^b, support springs S, cross-section areas A^S and A^b, and distances from the neutral axis to the interfaces C^s and C^b . In the symbols above as well as in the remainder of the text, the superscript "s" refers to the slab and the superscript "b" refers to the beam. The following quantities are related to the bars and are defined at the half-station: horizontal displacements U^S and $U^{b,}$ slip Y, shears V^S and V^b, shear connector modulus K^C, horizontal elastic springs K^S and K^b, distances from the neutral axes to the horizontal springs s^a and b^a and concentrated longitudinal loads and The quantities listed above are shown acting in the positive sense in Fig 6 interfaces and In the symbols above as well as in the remainder of the text, the superscript "s" refers to the slab and the superscript "b" refers to the beam.

4. DERIVATION OF EQUATIONS

Points are considered while deriving equations; The relationship between the horizontal displacement and the axial tension of the slab can be determined by examination; The axial tension is equal to the elongation multiplied by the axial spring Constant; differential equation is a moment-equilibrium equation. It also involves a summation of vertical forces and the moment-curvature relationship A free-body diagram of a portion of the system is shown; It should be noted that the applied torques and rotational restraints are felt by the system as transverse loads one station away from where the torque or rotational restraint is applied;

The nonlinearity occurs because the horizontal springs cause the final axial load distribution to be dependent on both vertical deflections and horizontal displacements; An iterative solution must be used when Equation nonlinear. In the iterative solution, the products of the horizontal spring constants and the horizontal displacements are computed and treated as known stiffness terms; Zero horizontal displacements are assumed for the first iteration. In each successive iteration, the horizontal displacements from the previous iteration are used. The process is continued until the computed displacements from two successive iterations agree to within specified tolerance linearity.

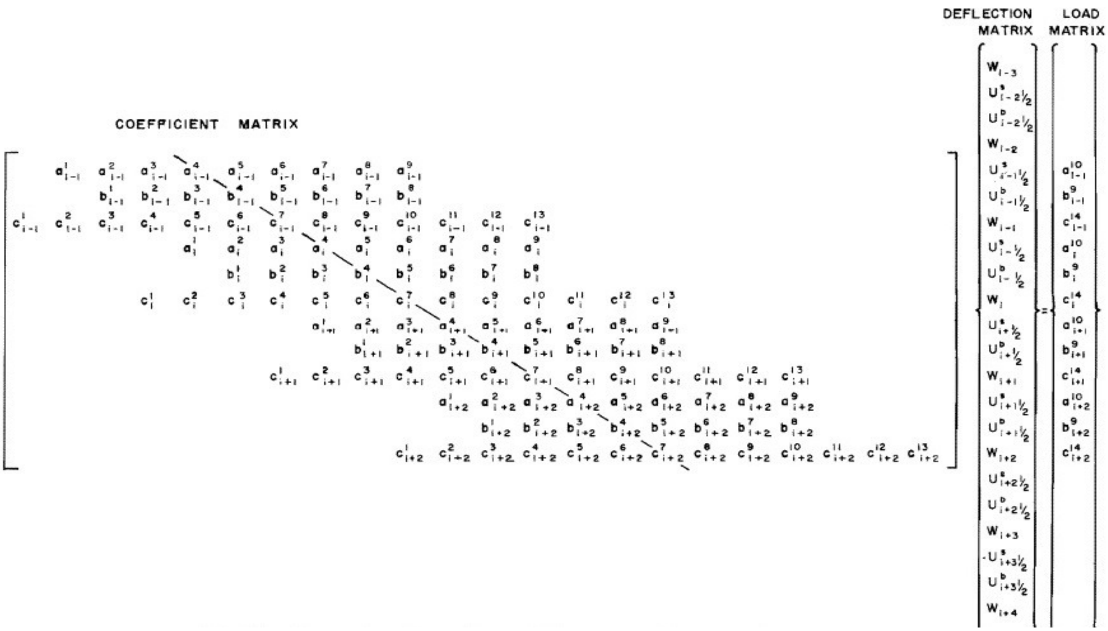


Figure 3

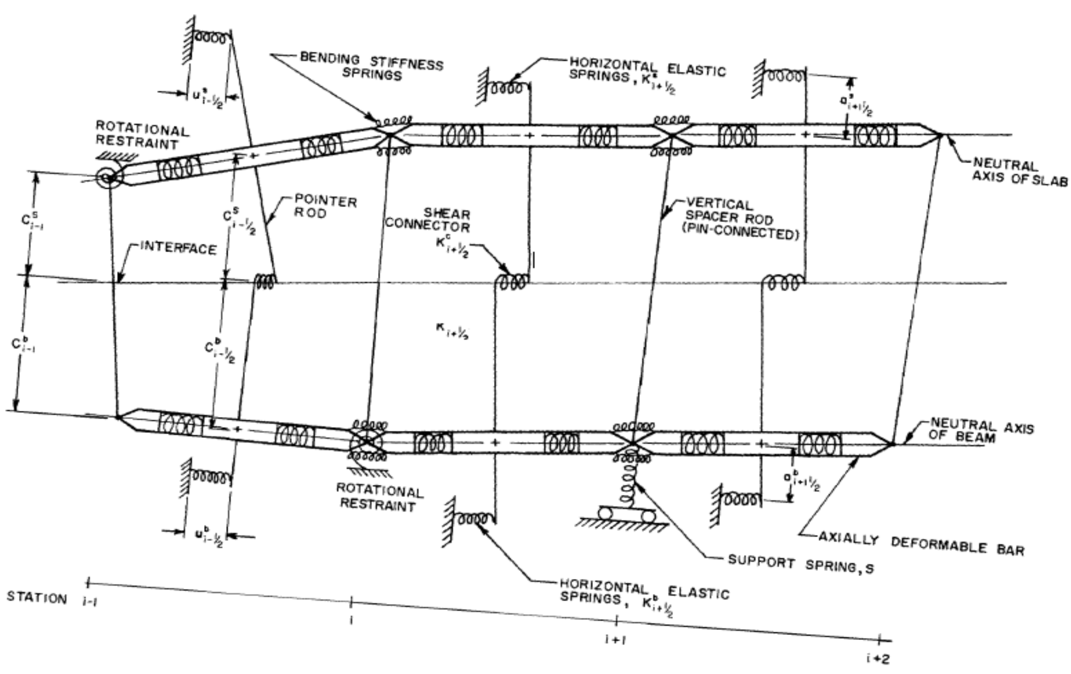


Figure 4

5. GOVERNING BOUNDARIES AND SPECIFIED CONDITIONS

A zero bending moment occurs at a point when the curvature and axial loads in a structure are both equal to zero. This condition is automatically created at each end of the structure. When Equation is written one station past the ends of the structure, some of the terms in the equation are equal to zero because physical properties of the system are zero past the ends. The remaining terms specify that the second derivative of vertical deflection with respect to distance (curvature) is equal to zero. A zero axial load is produced when the first derivative of horizontal displacement with respect to distance is set equal to zero.

A zero axial load is produced when the first derivative of horizontal displacement with respect to distance is set equal to zero. This condition is also created automatically by the physical properties of the system vertical deflection may be specified at any point in the structure by either of two methods. One method is to input a foundation spring of sufficient magnitude to insure a zero deflection. The other approach is to manipulate the matrix coefficients. A deflection can be specified at any Station i simply by the clearing of all of the coefficients in Equation to zero except C: which is set equal to 1.0 and 1. The desired deflection C: 4 which is set equal to 1.The (resistance to rotation) of a member may be controlled at any point by the specification of a rotational restraint. A rotational spring adds a bending moment to the system that is equal to the product of the slope at the point and the specified spring constant. A very rotational restraint cause S the slope at that point to be essentially zero. The zero curvature that is automatically created at the end of the member is over-ridden by the specification of a rotational restraint at the end. Horizontal displacements can be controlled by the specification of horizontal springs .No provision is made in the present analysis to control the displacements by manipulation of the matrix coefficients.

6. ACCURACY OF THE SOLUTION

Approximation errors are introduced when the finite-element model is substituted for the real structure. This type of error can be reduced to any desired level by dividing the model into more increments. An excessive number of increments should be avoided because computation time increases in simple proportion to the number of increments. Experience will enable the user to determine the optimum number of increments for his desired accuracy. Because of the large number of arithmetic operations involved in the solution, round-off errors may occur. Computer software using approximately 11 decimal digits has been used to verify the method of solution, and no significant errors have been observed in the practical problems that have been solved. Errors can be caused by the specification of unreasonably large values of certain of the physical properties. A good ru1e-of-thumb is that the magnitude of a rotational restraint should never be greater than 103 times the magnitude of the sum of the bending stiffness of the members at that station. Similarly, the shear connector modulus should not exceed 102 times the sum of the bending stiffness's.

7. RESULTS

After the vertical deflection and horizontal displacements have been computed, bending moment, axial load, slip, force per shear connector, shear, and support reaction can easily be determined. Bending moment is computed for the slab and beam by Equations derived. The force per shear connector is simply the product of the slip and the shear connector modulus. An expression for the shear in the slab is obtained by a summation of moments about the center of the slab bar in Fig 9a

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