Modelling and Design of LQG based Controller for Water Level of Nuclear Steam Generator

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Abstract: Stabilizing water level of the steam generator in nuclear power plant is a very important problem since its parameters vary with operating conditions and dynamics of the system is very different according to the power levels and changes as time goes on. In this paper, a concept of Linear Quadratic Gaussian (LQG) control scheme is proposed to achieve efficient control of steam generator. This approach employs a Kalman filter for state estimation and uses estimation error feedback to achieve offset free closed-loop behaviour. A linear fourth-order model has been simulated using MATLAB to carry out simulation studies.

Keywords: Nuclear power stations, steam generator model, water level control, LQG control

1. INTRODUCTION

Economic feasibility of a nuclear power plant requires smooth and uninterrupted plant operation in the face of varying electrical power demand. Unplanned shutdowns or reactor trips initiated due to conservative safety considerations, which in turn are necessitated by poor control, are particularly expensive and must be minimized. The water level regulation of SG is difficult control problem due to its nonlinear behaviour, non-minimum phase characteristics, unstable plant dynamics and unreliable sensor feedback at low power. Too high water level produces wet steam which can damage the turbine blades. Too low water level causes poor cooling of the nuclear reactor, which ultimately results in reactor trips. In this work, it is desired to control a nuclear steam generator system using a linearized mechanistic model. LQG control scheme is developed to achieve efficient control of SG. The task of a control scheme is to respond quickly to power load (i.e. steam demand) changes. A major advantage of LQG controller is that it can handle multi-variable interactions and can be designed for open loop unstable plants relatively easily.

Factors Leading to Poor Control:

The difficulties in designing an effective level control system for the SG arise from a number of factors:

- *Nonlinear plant characteristics*. The plant dynamics are highly nonlinear. This is reflected by the fact that the linearized plant model shows significant variation with operating power.

- *Non minimum-phase plant characteristics*. The plant exhibits strong inverse response behaviour, particularly at low operating power due to the so-called "swell and shrink" effects. This non minimum phase characteristic limits the achievable controller bandwidth.
- *Sensor measurements*. At low powers, the flow measurement sensors are known to be unreliable and this precludes effective use of feed forward control.
- *Constraints*. The feed-water system can only deliver a limited throughput of water to the SG. This imposes a *hard* limitation on the available control action, and thus on the available controller bandwidth. Moreover input constraints can lead to the classical controller windup problem [13] which causes degradation of system performance and sometimes even instability if not accounted for in the controller design.

2. MODELLING OF U-TUBE STEAM GENERATOR

The parameter-dependent transfer function model identified by Irving *et al.* (1980) [1] is the most widely accepted SG models for use in controller design.

Let *y1* and *y2* be the narrow and wide range water levels *L1* and *L2*, and let *w* and *d* be the feed-water and steam flow-rates. Then, the transfer functions relating the inputs *w* and *d* to the water levels *y1* and *y2* are given by:

$$Y(S) = \frac{G_1}{s}\left(q_w(s) - q_v(s)\right) - \frac{G_2}{1+\tau_2 s}\left(q_w(s) - q_v(s)\right) + \frac{G_3 s}{\tau_1^{-2} + 4\pi^2 T^{-2} + 2\tau_1^{-2} s + s^2} q_w(s)$$

Where,

$Y(s)$, $q_w(s)$, and $q_v(s)$ are water level, feed-water flow-rate, and steam flow-rate respectively,

$\frac{G_1}{s}$ is the mass capacity effect of the UTSG. It integrates the flow difference to calculate the change in water level. This term accounts for the level change due to feed water inlet to steam generator and the steam outlet from it. This quantity means the actual water capacity which critically affects the removal capability of the primary heat. G_1 is a positive constant and does not depend on load. $\frac{G_2}{1+\tau s}$ is the thermal negative effect caused by "swell and shrink". Since these phenomena exhibit exponential responses for step changes of the feed water flow-rate and the steam flow-rate, they are described by a first-order equation. G_1 is positive and dependent on load. As load increases G_2 decreases. The third term is the mechanical oscillation effect caused by the inflow of the feed-water to the UTSG. This is a mechanical oscillation term due to momentum of the water in the downcomer. All the water removed from the steam is returned to the downcomer and is recirculated. The recirculating water has large momentum acting against relatively small flow-rate changes. When the feed-water flow-rate is suddenly decreased, the water level in the down comer falls initially and then begins to oscillate. This is due to the momentum of the water in the down comer keeping the recirculating flow going down initially and then slowing down. The mechanical oscillation disappears completely after a small multiple of the damping time constant. The variable G_3 is positive.

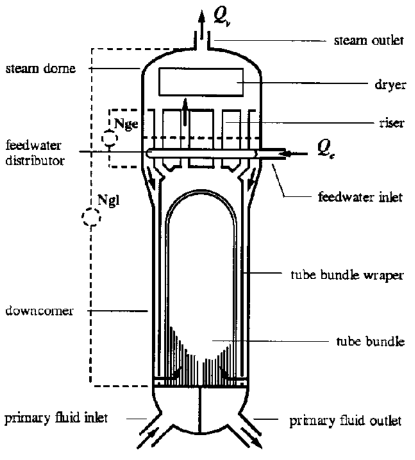

Fig.1 Schematic of UTSG

The state equations are defined as follows:

$$\delta\dot{x}_1(t) = G_1\big(\delta q_w(t) - \delta q_v(t)\big)$$

$$\delta\dot{x}_2(t) = -\tau_2^{-1}\delta x_2(t) - \frac{G_2}{\tau_2}\big(\delta q_w(t) - \delta q_v(t)\big)$$

$$\delta\dot{x}_3(t) = -2\tau_1^{-1}\delta x_3(t) + \delta x_4(t) + G_3\delta q_w(t)$$

$$\delta\dot{x}_4(t) = -(\tau_1^{-2} + 4\pi^2 T^{-2})\delta x_3(t)$$

and the output (water level) is

$$\delta y_p(t) = \delta x_1(t) + \delta x_2(t) + \delta x_3(t)$$

If we define,

$$\delta x_p(t) \triangleq [\delta x_1\ \delta x_2\ \delta x_3\ \delta x_4]^T$$

the dynamics of the steam generator system can then be reduced to the following state-space equations:

$$\delta \dot{x}_p(t) = A_p \delta x(t) + B_p \delta q_w(t) + F_p \delta q_v(t)$$

$$\delta y_p(t) = C_p \delta x(t)$$

where *Ap*, *Bp*, and *Cp* matrices are given as:

$$A_P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & 0 \end{bmatrix}, B_P = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{bmatrix}, F_P = \begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 0 \end{bmatrix}, C_P = [1 \quad 1 \quad 1 \quad 0]$$

Where

$a_{22} = -\tau_2^{-1}, a_{33} = -2\tau_1^{-1}, a_{34} = 1, a_{43} = -(\tau_1^{-2} + 4\pi^2 T^{-2})$

$b_1 = G_1, b_2 = -G_2\tau_2^{-1}, b_3 = G_3$

$d_1 = G_3, d_2 = \dfrac{G_2}{\tau_2}$

The dynamic parameters with respect to operating power linearized at different power level is shown in the table below:

Table 1. Dynamic parameters

$\boldsymbol{q_v}$ $(\boldsymbol{kg/s})$	57.4	180.8	381.7	660	1435
P (%power)	5	15	30	50	100
$\boldsymbol{G_1}$	0.058	0.058	0.058	0.058	0.058
$\boldsymbol{G_2}$	9.63	4.46	1083	1.05	0.47
$\boldsymbol{G_3}$	0.181	0.226	0.310	0.215	0.105
$\boldsymbol{\tau_1}$	41.9	26.3	43.4	34.8	28.6
$\boldsymbol{\tau_2}$	48.4	21.5	4.5	3.6	3.4
T	119.6	60.5	17.7	14.2	11.7

3. CONTROLLER DESIGN

Linear Quadratic Gauusian: (LQR) technique is applied to design an optimum controller that forces the plant output water level to follow a desired water level.

The controller design lie on finding the *u(t)* control vector that minimize the following cost functional:

$$J = \int_{t_0}^{\infty} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt$$

where Q and R are constant weighting matrices; the state weighting matrix Q must be symmetric and at least positive semi-definite and the control weighting matrix R is selected to be symmetric and positive definite.

The optimal control *u(t)* is generated from the state perturbation *x(t)* by a linear constant gain feedback:

$$u(t) = -Kx(t)$$

where K is a constant feedback gain matrix given by $K = R^{-1}B^T P$, and P is the solution of the algebraic matrix Riccati equation $PA - A^T P - Q + PBR^{-1}B^T P = 0$.

The existence and uniqueness of solution for the above equation are guaranteed by the following assumptions: *(A,B)* is a controllable pair and *(A,*$Q_{1/2}$*)* is an observable pair.

Observer design:

The implementation of the controller requires that the full state of the system is available. We like to estimate the states from the output measurements. By Kalman filter the optimal estimate $\widehat{x_p}(t)$ can be generated by

$$\dot{\widehat{x_p}}(t) = A_0\widehat{x_p}(t) + B_0 u_{pq}(t) + L[y_{pq}(t) - C_0\widehat{x_p}(t)]$$

The filter gain matrix L is given by

$$L = P_2 C_0^T R_2^{-1}$$

Where P_2 is the solution of the algebraic Ricatti equation

$$A_0P_2 + P_2A_0^T + Q_2 - P_2C_0^T R_2^{-1}C_0P_2 = 0$$

The observer state equation can be reduced to

$$\dot{\widehat{x_P}}(t) = (A_0 - B_0K_1 - LC_0)\widehat{x_P} + LC_P x_P$$

Where K_1 is the gain required ensure output tracking derived before.

4. RESULT

Simulation results are provided to validate the effectiveness of the designed controller as shown below:

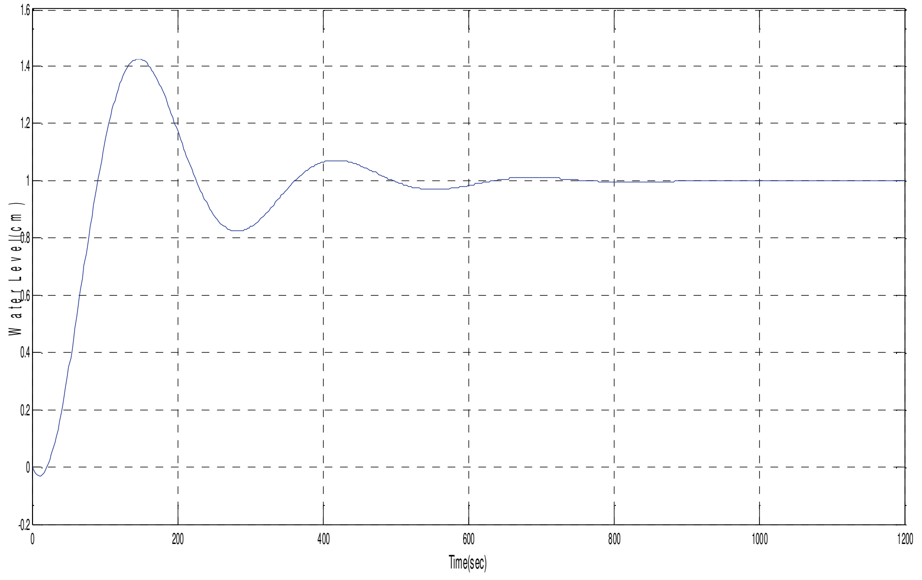

Fig. 2 SG water level response for LQG controller

5. CONCLUSION

Control of UTSG water level strongly affects nuclear power station availability. There has been a special interest in this problem during low power transients because of the dominant thermal dynamic effects known as shrink and swell. Also, the non-minimum phase property, changing parameter according to power level, make it difficult to control the water level of SG to control the water level of a steam generator. In this paper, feed water controller design is made by LQR method to consider the constraints on both states and input energy. On account of uncertainties on system and measurement noises, an observer is constructed by using Kalman filter. The simulations demonstrated the effectiveness of the proposed controller in diverse operating conditions. The LQG controller shows an improvement in water level set point tracking and an increased ability in disturbance (in the form of steam flow rate changes) rejection.

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