# Stability Analysis of Fuzzy Model-Based Nonlinear Controller for Nonlinear System

**Suman Lata**
*Department of Electrical Engineering*
*Aligarh Muslim University, Aligarh , Aligarh-202002, India*

***Abstract: This paper presents the stability analysis of a fuzzy-model-based nonlinear control system consisting of a nonlinear plant and a nonlinear controller. First, we represent a nonlinear plant with Takagi-Sugeno type fuzzy model. In this type of model, local dynamics in different state space are represented by linear models. The overall model of the system is achieved by fuzzy blending of these linear models. Then, we use the concept of Parallel Distributed Compensation (PDC) to design fuzzy controllers to stabilize fuzzy system. The idea is to design a compensator for each rule of fuzzy model. For each rule, we can use linear control design techniques. The resulting overall controller, which is nonlinear, is a fuzzy blending of each individual linear controller. Stability conditions are given to guarantee the stability of fuzzy model-based control system. The results obtained are illustrated with the examples.***

***Keywrods: Fuzzy plant model, nonlinear controller, nonlinear plant, stability analysis, parallel distributed compensation.***

## 1. INTRODUCTION

There have been many successful applications of fuzzy control. Stability theory will certainly give us a wider view on the future development of fuzzy control. Many researchers have worked to improve the performance of the fuzzy logic controller and ensure their stability in [3]-[5]. Despite the success, it has many basic issues related to stability analysis and systematic designs of fuzzy control system have to be addressed. This paper deals with the detailed of a systematic design methodology for fuzzy control of a class of nonlinear systems. There are several approaches to control nonlinear system. A typical approach is the feedback stabilization of nonlinear systems where a linear feedback control is designed for the linearized the system about a nominal operating point. This approach only causes a local result. In this paper, we consider a nonlocal approach which is conceptually simple and straightforward. Feedback stabilization can utilize linear feedback control techniques. The procedure is as follows [6]: first the nonlinear plant is represented by a Takagi-Sugeno type fuzzy model. In this model, local dynamics in different state space regions is represented by linear models. The overall model of the system is achieved by fuzzy blending of these linear models. For control design, we used fuzzy model called parallel distributed compensation scheme. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller, which is nonlinear in general, is again a fuzzy blending of each individual linear controller. This paper also discusses stability analysis of nonlinear systems. Stability conditions of both fuzzy models and fuzzy control systems are given in this paper. The design methodology is illustrated by examples [6].

Stability analysis of Takagi-Sugeno fuzzy models and control design problems via parallel distributed compensations are presented in Section II; Simulation results implemented in MATLAB are discussed in Section III; and finally, the conclusions are given in Section IV.

## 2. STABILITY ANALYSIS AND PARALLEL DISTRIBUTED COMPENSATION

This paper only deals with the description of results for discrete-time systems given in [6]. In the proposed procedure, we represent a given nonlinear plant by Takagi-Sugeno fuzzy model. The system dynamics is captured by a set of fuzzy implications which characterize local relations in the state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy blending of the linear system models.

Specifically, the Takagi-Sugeno fuzzy system is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of a system. The $i^{th}$ rule of the fuzzy system is of the following form:

Rule $i^{th}$: IF $x_1(k)$ is $M_{i1}$ …… and $x_n(k)$ is $M_{in}$ THEN $x(k+1) = A_i x(k) + B_i u(k)$

Where
$$x^T(k) = [\, x_1(k), x_2(k) \;....\; x_n(k)\,]$$
$$u^T(k) = [\, u_1(k), u_2(k) \;...\; u_m(k)\,]$$

i = 1, 2, 3… r. and r is the number of IF-THEN rules. $M_{ij}$ are fuzzy sets. Given a pair of $(x(k), u(k))$, the final output of the fuzzy system is defined as follows:

$$x(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\{A_i x(k) + B_i u(k)\}}{\sum_{i=1}^{r} w_i(k)} \tag{1}$$

Where $w_i(k) = \prod_{j=1}^{n} M_{ij}(x_j(k))$.

$M_{ij}(x_j(k)$ is the grade of membership of $x_j(k)$ in $M_{ij}$.

The open loop system of (1) is

$$x(k+1) = \frac{\sum_{i=1}^{r} w_i(k) A_i x(k)}{\sum_{i=1}^{r} w_i(k)} \tag{2}$$

Where it is assumed that

$$\sum_{i=1}^{r} w_i(k) > 0$$

$$w_i(k) \geq 0 \qquad i = 1,2,....,r$$

For all k, each linear component $A_i x(k)$ is called a subsystem.

### *A. Parallel Distributed Compensation*

We utilize the concept of parallel distributed compensation (PDC) to design a fuzzy controller to stabilize fuzzy system. The idea is to design a compensator for each rule of the fuzzy model. Fig. 1 shows the design of PDC. For each rule, we can design a controller with the help of linear control design techniques. The resulting overall fuzzy controller, which is nonlinear, is a fuzzy blending of each individual linear controller. The fuzzy controller shares the same fuzzy sets with that of the fuzzy system (1). The $i^{th}$ rule of the fuzzy controller is given by:

Rule $i^{th}$: IF $x_1(k)$ is $M_{1i}$ …… and $x_n(k)$ is $M_{ni}$ THEN $u(k) = -F_i x(k)$

Where i=1, 2, 3… r. Hence, the fuzzy controller will have overall output as:

$$u(k) = \frac{-\sum_{i=1}^{r} w_i(k) F x(k)}{\sum_{i=1}^{r} w_i(k)} \tag{3}$$

Substituting (3) into (1) we obtain

$$x(k+1) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)\ w_j(k) \left\{A_i - B_i\ F_j\right\} x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)\ w_j(k)} \tag{4}$$

**Theorem 2:** The equilibrium of a fuzzy control system (4) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$\{A_i - B_i F_j\}^T P \{A_i - B_i F_j\} - P < 0$$

For $w_i(k) \cdot w_j(k) \neq 0, \forall k, \quad i, j = 1,2,...,r$ (5)

Note that system (4) can be also written as

$$x(k+1) = \frac{1}{W}\left[\sum_{i=1}^{r} w_i(k)\ w_j(k)\{A_i - B_i F_i\}x(k) + 2\sum_{i<j} w_i(k)\, w_j(k)\ G_{ij}\ x(k)\right] \quad (6)$$

Where $G_{ij} = \frac{\{A_i - B_i F_j\} + \{A_j - B_j F_i\}}{2}, \quad i < j; \qquad W = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)\ w_j(k)$

Therefore, we have the following sufficient condition.

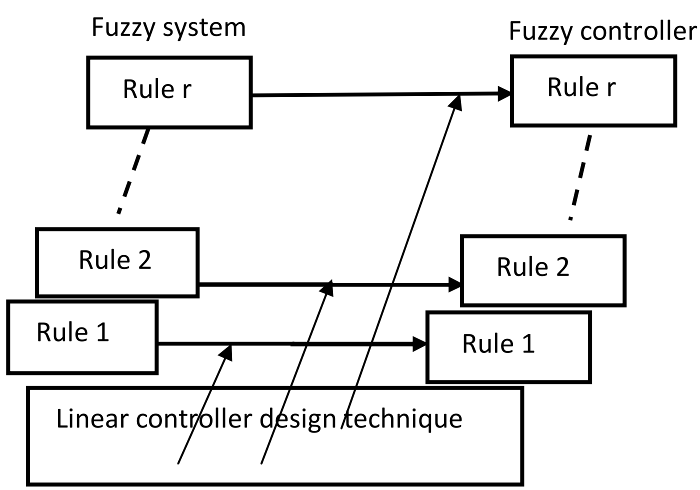


**Fig. 1: Parallel distributed compensation (PDC) design**

**Theorem 3:** The equilibrium of a fuzzy control system (4) is asymptotically stable in the large if there exists a common positive definite matrix P such that the following two conditions are satisfied.

$$\{A_i - B_i F_j\}^T P \{A_i - B_i F_j\} - P < 0, \quad i = 1, 2, ..., r \tag{7}$$

$$G_{ij}^T P G_{ij} - P < 0, \qquad i < j \leq r \tag{8}$$

The control design problem is to select $F_i$ $(i = 1, 2, ...., r)$ such that conditions (7) and (8) in theorem 3 are satisfied.

Using the notation of quadratic stability, we can also think of the control design problem as finding $F_i$'s such that the closed loop system (4) is quadratically stable. If there exist such $F_i$'s, the system (1) is also said to be quadratically stablizable via PDC design. In general, we design a controller for each rule and check whether the stability conditions are satisfied.

Assume $(A_i, B_i)$ are controllable. If $B_i = B\,(i = 1,2,...., r)$ and we choose $F_i$ such that

$$A_i - B F_i = G \tag{9}$$

Where G is a Hurwitz matrix. There exists a P such that

$$G^T P G - P < 0 \tag{10}$$

Because of (9) and

$$G_{ij} = G; \qquad i \leq j$$

The following theorem follows from Theorem 3.

**Theorem 4:** In the case of $B_i = B,\ i = 1, 2, ...., r$ the equilibrium of fuzzy control system (4) can be made asymptotically stable in the large by fuzzy PDC controller (3) where $F_i$ satisfies (9).

## 3. SIMULATION RESULTS

**Example 1:** Let us consider the fuzzy system with two rules as:

Rule 1: IF $x_2(k)$ is $M_1$ THEN $x(k+1) = A_1 x(k) + B\, x(k)$

Rule 2: IF $x_2(k)$ is $M_2$ THEN $x(k+1) = A_2 x(k) + B\, x(k)$

Where $A_1 = \begin{bmatrix} 1.2 & -0.65 \\ 1.5 & 0 \end{bmatrix}$; $A_2 = \begin{bmatrix} -1.2 & -0.65 \\ 1.5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$A_1$ and $A_2$ are stable, in other words, the linear subsystems are stable. Fig. 2 shows the membership functions of $M_1$ and $M_2$. We use the PDC controller given in (3) for fuzzy controller and choose the closed loop Eigen values to be [0.35 0.4], we obtain

$F_1 = [0.45 \quad -0.5567]; F_2 = [-1.95 \quad -0.5567];$

And closed loop gains: $A_1 - BF_1 = A_2 - BF_2 = G = \begin{bmatrix} 0.75 & -0.0933 \\ 1.5 & 0 \end{bmatrix}$

G is positive definite matrix. So G is stable. If we choose the positive definite matrix P to be

$$P = \begin{bmatrix} 7.145 & -3.5725 \\ -3.5725 & 8.93 \end{bmatrix}$$

Stability condition given in (10) is satisfied. Hence the closed loop system $x(k+1) = G\,x(k)$ becomes stable. The implementation of fuzzy control system is done in MATLAB. Fig. 3 illustrates the behaviour of the fuzzy control system for the initial condition $x = [0.90 \quad -0.70]^T$.

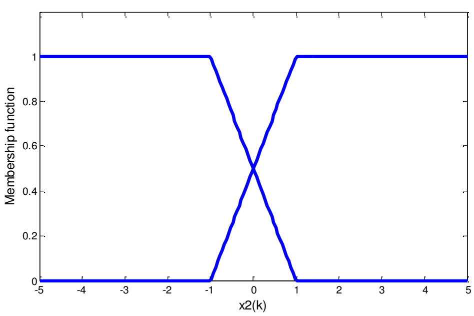


**Fig. 2: Membership function**

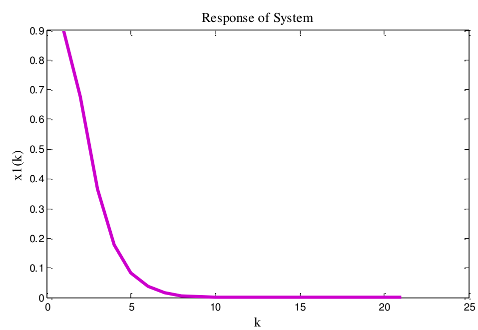


**Fig. 3: Response of example 1**

**Example 2:** Let us consider the fuzzy system with two rules given by:

Rule 1: IF $x_2(k)$ is $M_1$ THEN $x(k+1) = A_1 x(k) + B_1 x(k)$

Rule 2: IF $x_2(k)$ is $M_2$ THEN $x(k+1) = A_2 x(k) + B_2 x(k)$

Where $A_1 = \begin{bmatrix} 2 & -0.35 \\ 1.5 & 0 \end{bmatrix}$; $A_2 = \begin{bmatrix} -2 & 0.35 \\ 1.5 & 0 \end{bmatrix}$

$B_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$; $B_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

The membership functions of Example 1 are used in the simulation. Again choose the closed-loop Eigen values to be [0.4 0.35], we have following gains.

$F_1 = [-1.0364 \quad 0.2136]$; $F_2 = [0.8685 \quad -0.1444]$;
And closed loop gains can be found as:

$$A_1 - B_1F_1 = G_1 = \begin{bmatrix} 0.9636 & -0.1364 \\ 2.5364 & -0.2136 \end{bmatrix}$$
$$A_2 - BF_2 = G_2 = \begin{bmatrix} 0.6056 & -0.0832 \\ 0.6315 & 0.1444 \end{bmatrix}$$
$$G_{12} = \begin{bmatrix} -1.1203 & 0.2483 \\ 1.5839 & -0.0346 \end{bmatrix}$$

$G_{12}$is positive definite matrix. So $G_{12}$ is stable. If we choose the positive definite matrix P to be

$$P = \begin{bmatrix} 2.4194 & 4.0502 \\ 4.0503 & 12.7573 \end{bmatrix}$$

The stability conditions are satisfied which are:

$$\{A_i - B_i F_j\}^T P \{A_i - B_i F_j\} - P < 0, \quad i = 1, 2, ..., r$$

$$G_{ij}{}^T P \ G_{ij} - P < 0, \qquad i < j \leq r$$

In other words, the closed loop system which consists of the fuzzy model and PDC controller is globally asymptotically stable. The implementation of fuzzy control system is done in MATLAB. Fig. 4 illustrates the response of the fuzzy model based fuzzy control design methodology.

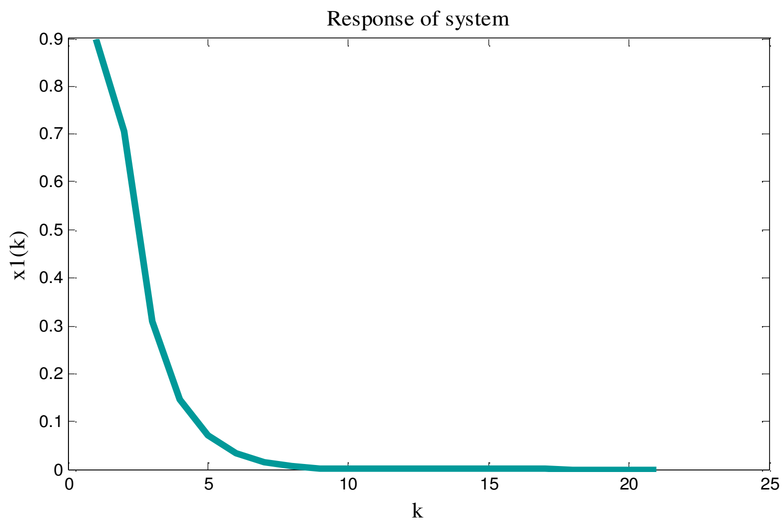


**Fig. 4: Response of Example 2**

## 4. CONCLUSIONS

The stability of fuzzy model based controller for nonlinear system has been analysed. The design methodology based on Takagi-Sugeno fuzzy model and parallel distributed compensation control design has been presented. Stability conditions of fuzzy models and fuzzy control systems have been discussed. Examples have given to show the stability of fuzzy control system. The implementation of fuzzy control system in examples was done in MATLAB.

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