Dynamics of Coupled FN Neuron Model

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ABSTRACT

In this paper, we will discuss the effect of coupling on the neuron dynamics. We will begin by analyzing the dynamics of two neurons using the FN model with a constant coupling. Coupling of neurons signifies simulation of neural networks, of course in the most basic way possible. The parameter that we change is the input current. Below a particular value of I the system returns to the fixed point. Our next problem is to consider 3 neurons and with the same coupling. We vary the coupling strength. We also explore the circular coupling. The neurons are, in reality, not linked to each other in such a simplified manner. But to get an idea of the dynamics, it is good enough to consider a simplified model. Mathematically, the FN model refers to two coupled nonlinear ODE. The dependent variables under consideration are the membrane potential of the neurons and the recovery variable of the neuron membrane potential. The recovery variable represents some kind of a relaxation time. Soon after a neuron fires a certain minimum time called the refractory period is required before it can fire again. Also it is very important to note that a neuron does not fire i.e. it does not send a signal below a threshold value of the input current.

The system can be treated as a dynamical system which is nonlinear. For such a system, we investigate the bifurcations as the strength of coupling is changed. In this paper we will study the effect of adding more neurons on the dynamics of the system. In particular we will be extending the study by Zechariah Thurman where the dynamics of uncoupled neuron and two coupled neurons have been investigated . The Fitzhugh-Nagumo neuron model has been considered.We will investigate the dynamics associated with coupling three neurons. The stimulation is through an input current to one of the neurons. We study the effect of varying this input parameter on the dynamics of the system. The effect of adding a single neuron to the network of neurons does alter the dynamics of the system. Our observation is that the threshold for firing becomes higher with increase in the dimensionality of the system. Also the threshold shows an increase with higher constants . Similar results are obtained for circular coupling which also delays the onset of firing as compared to the linear coupling.

Keywords: Fitzhugh-Nagumo model, dynamics, threshold current, coupled neurons

1. INTRODUCTION

The brain is an extremely complex information processing system having about 10^{11} neurons . Each neuron is connected to thousands of neurons[1] and therefore communications through these neurons is an extremely important field . If a neuron is stimulated by a pulse of current, it is observed that it fires i.e. it produces an output signal provided the input is above a threshold value.

In this paper, we consider the simplified Fitzhugh Nagumo model for the neurons[2, 3] which manages to capture the essential dynamics of the neurons and therefore we use this model to investigate the dynamics[4] of three coupled neurons. We use MATLAB to numerically solve the six coupled nonlinear differential equations [5, 6]. These differential equations represent the variation of membrane potentials of all the three neurons and represent the rate of change of the refractory period for these neurons. The refractory period just represents some kind of relaxation time that the neuron needs before it can fire again when an input above the threshold is injected. While studying the dynamics of the neurons we reproduce the results of Zechariah Thurman[7]. We extend their model to a system of 3 neurons. We study the effect of adding a neuron to the dynamics of the system. Addition of the third neuron implies an increase in the dimension of the system. Solving numerically, the system of six nonlinear coupled differential equations, reveals that the threshold for the input current for the neurons to fire, increases. We also change the value of the coupling constant and discover that the increase in the coupling constant has an influence on the dynamics of the system. We study the features of this dynamics and in particular concentrate on bifurcation points for two kinds of coupling linear, as well as circular.

2. DIFFERENTIAL EQUATIONS

Most biological phenomena are represented by differential equations so that they can be described at any instant of time and not at isolated times. With this as the background, , the state of the physical system of a single neuron, two neurons and three neurons will be described by differential equations . The FN model equations are derived from the Hodgin Huxley model and are represented by two variables $v\ and\ w$ representing the membrane potential and the recovery variable for the membrane potential for both slow and fast ionic currents, respectively [2, 3, 8].

For the single FN neuron the differential equations are [7]

$$\dot{v} = c\left(v - \left(\frac{1}{3}\right) v^3 + w + I \right). \tag{1}$$

$$\dot{w} = -\left(\frac{1}{c}\right)(v - a + bw) \tag{2}$$

ISBN: 978-93-83083-88-6

Where the constants have the following values $a = 0.75; b = 0.8; c = 3.0$
I is the input current injected.

The dynamics of this system is well understood [7] . We next consider the differential equations for a system of two neurons. We consider them to be coupled. We inject the input current in the first neuron and study its effects. Our ultimate aim is to consider the FN model for three neurons. We realize that for linear coupling, our second neuron can be coupled both to the first and the third neuron. Besides the linear coupling, we will extend our work to considering circular coupling of the three neurons. A brief comparison of the dynamics between the two types of coupling will be taken up. In reality, we have huge networks of neurons so studying the two kinds of coupling will be a step forward in understanding the dynamics. We conclude this section by setting up the differential equations for the three neurons for linear coupling. This will be a system of six differential equations, three for the three neuron membrane potentials and three for the their recovery variables.We will subsequently add a couple of terms for the circular coupling. In our simplest model, we will continue to inject the input current in the first neuron and study the effect on all the three neurons. Just to simplify the model we consider the strength of the coupling to be a constant for all the neurons. In reality this may not be true but it helps us in understanding the essential dynamics of the system.

$$\dot{v}_1 = c[v_1 - \left(\tfrac{1}{3}\right)(v_1)^3 + w_1 + I + k(v_2 - v_1)]. \quad (3)$$

$$\dot{w}_1 = -\left(\tfrac{1}{c}\right)[v_1 - a + b(w_1)] \quad (4)$$

$$\dot{v}_2 = c[\, v_2 - \left(\tfrac{1}{3}\right)(v_2)^3 + w_2 + k(v_1 - v_2) + k(v_3 - v_2)] \quad (5)$$

$$\dot{w}_2 = -\left(\tfrac{1}{c}\right)[v_2 - a + b(w_2)] \quad (6)$$

$$\dot{v}_3 = c[v_3 - \left(\tfrac{1}{3}\right)(v_3)^3 + w_3 + k(v_2 - v_3)] \quad (7)$$

$$\dot{w}_3 = -\left(\tfrac{1}{c}\right)[v_3 - a + b(w_3)] \quad (8)$$

In these expressions k represents the coupling constant, a, b, c have the same values as in [7] and I represents the injected input current.

3. ANALYSIS OF THE DYNAMICS

For nonlinear oscillators with certain kinds of coupling we find that the oscillators can be completely synchronized to a common frequency. We refer to fig 1. This is the graphical representation of the dynamics of two neurons which are coupled and have 0 input current. In fig2, after the threshold value of 0.4913 for I, we find that both the neurons starts firing but neuron 1

ISBN: 978-93-83083-88-6

which has the input injected into it fires a stronger signal. Our objective in this paper is to explore the dynamics presented by three coupled neurons. Similar to reference [7] we will begin by taking the coupling constant as 0.113 . Fig 3 represents the variation of membrane potentials of neurons with respect to time when the input current is below the threshold value. The dynamics gets richer as we change the value of the input current above the threshold value.This is depicted in fig 4. The bifurcation point for the firing of neurons is 0.49679. At this point the three neurons fire in complete synchronization .Once again as in the two neuron system we find that the signal strength is highest for neuron 1.

In Fig 5 we begin with exploring the dynamics of three neurons with circular coupling with the input current below the threshold value. By plotting the membrane potentials vs time, we detect the threshold value of current which is the bifurcation point. Fig 6 represents firing of neurons just above the threshold value of I. Instead of three curves only two are visible. This is because there is absolute synchrony between the dynamics of the second and third neuron under circular coupling. The important comment is that viewing these plots aids us in determining the threshold value for the membrane potential to change from a fixed value to a periodic variation. Since the aim is also to compare the dynamics of the three neuron system under different strengths of coupling we have studied this dynamics by changing the coupling constants. These are depicted in figures 7-10.

4. RESULTS AND CONCLUSIONS

In this paper we have studied the dynamics of a system of three neurons by considering the FN neuron model. We have determined the threshold values of the input current at which the neurons' membrane potential changes from a constant value to a periodic form. The effect of addition of a neuron on the dynamics is very well depicted in the solutions to the given differential equations. Comparing our results with the system of two neurons [2], we find that adding a third neuron to the system delays the onset of neurons' firing, meaning that the neuron now fires at a higher value of the input current. This is similar to the transition from the a single neuron system to a two neuron system [7]. We have also studied the dynamics of the system of the three neurons from linear to a circular coupling. Circular coupling is an important factor since in reality the neurons have an extremely complex network and therefore investigation of the dynamics of such a system is significant for understanding the processes of neuron interactions and information coding. An important observation is that under the circular coupling, the two neurons which are coupled to the neuron which has the input current injected into it is in complete synchrony with each other. They follow exactly the same dynamics. This feature is completely absent in the case of the linear coupling where all the three neurons have different membrane potentials. For the circular coupling we have also altered the coupling constant and studied the effect on the threshold of input current which results in the bifurcation of the system. As we increase the coupling constant we find that the threshold value for the transition from the stable state to the periodic state starts at a higher value of the input current. Our second conclusion is that as compared to the linear coupling with the circular coupling, neurons start firing at a higher value of I. When we couple neuron 1 and 3 besides coupling 2 and 3, we find that the membrane potentials of the three neurons are almost of the same strengths .

We conclude this section with scope for future work in this field. An interesting problem could be to explore the dynamics of a system with larger dimensions which can happen by considering four or more neurons. The other area which has future scope is considering the dynamics using delayed differential equations since there is a time lag between injecting the input current and the production of an output signal by the neuron . We could also describe a system with both forms of coupling and considering a time varying input current. All these investigations will certainly be a step forward in understanding essential features of the dynamic neurons.

5. ACKNOWLEDGEMENT

We thank the Department of Biotechnology for funding this project under the star college scheme. We are extremely grateful to the NGO, Krishi Sanskriti for providing us the platform for presenting our work.

2 NEURONS

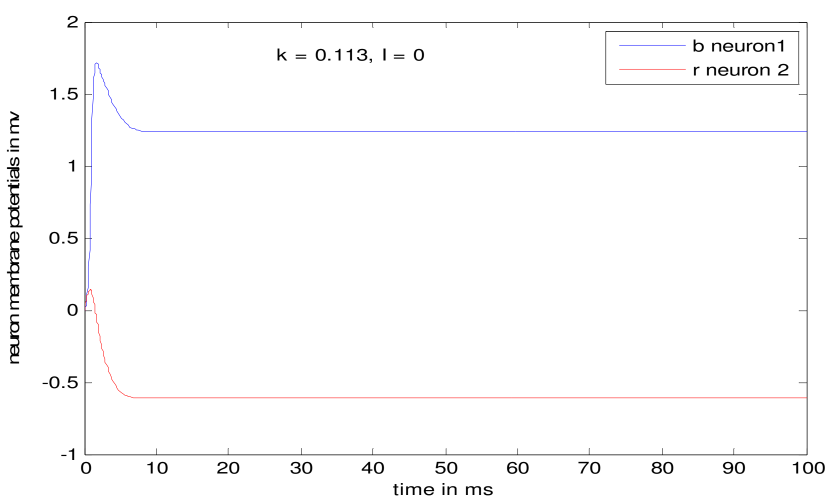

Fig 1: plot of neuron membrane potential with time with I=0

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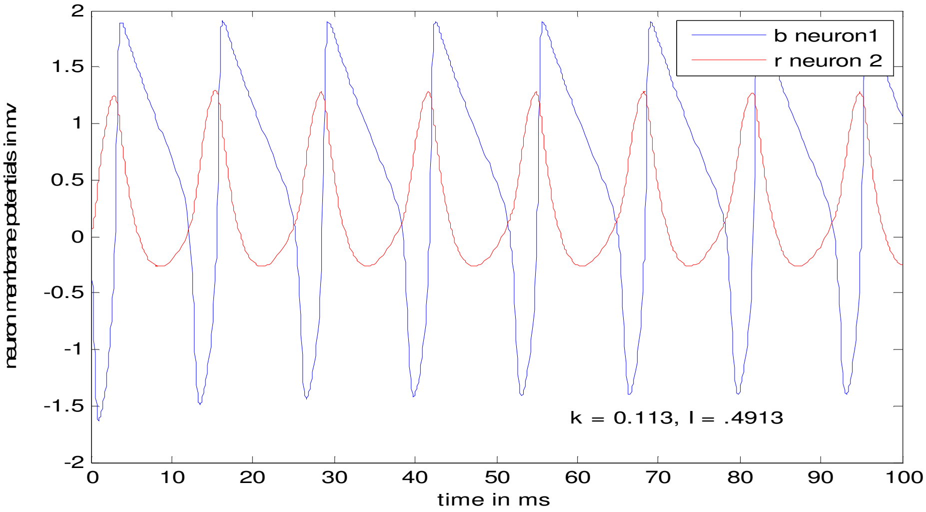

Fig 2: 2 neurons ;plot of neuron potential with time with I =0.4913

Linear coupling with three neurons

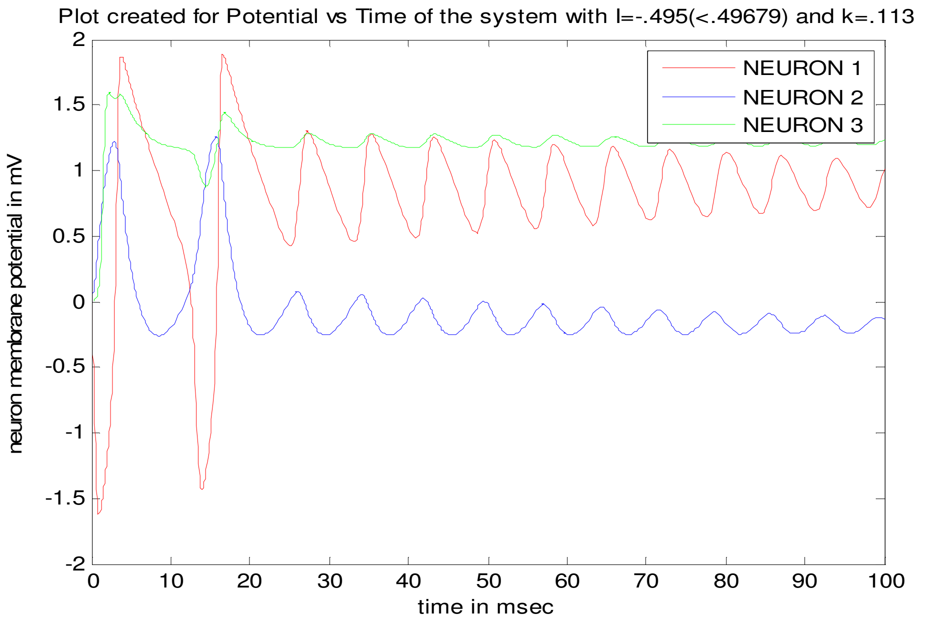

Fig 3:

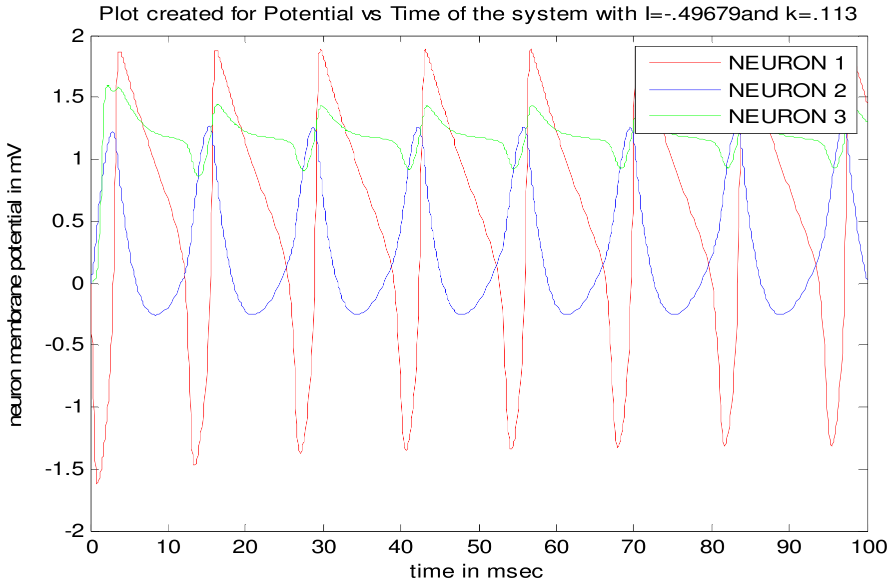

Fig 4: 3neurons with I > threshold

Circular coupling with 3 neurons

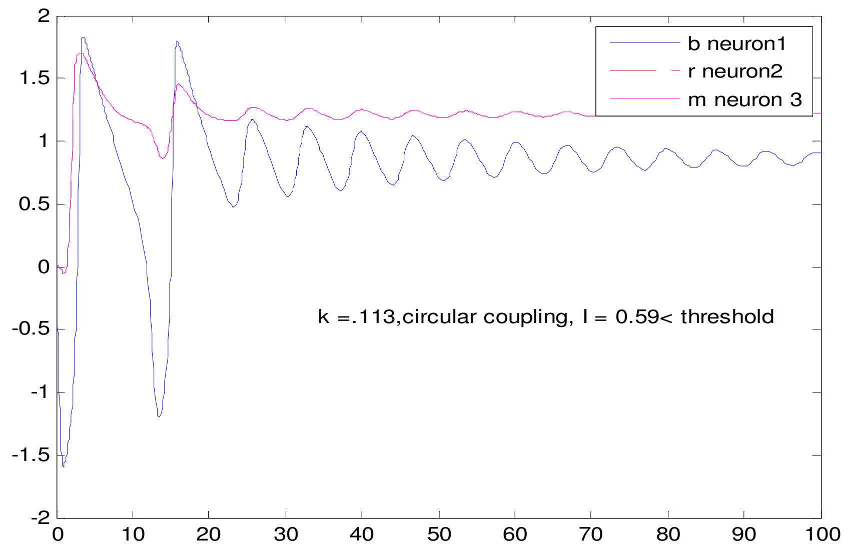

Fig 5: 3 neurons with I < threshold

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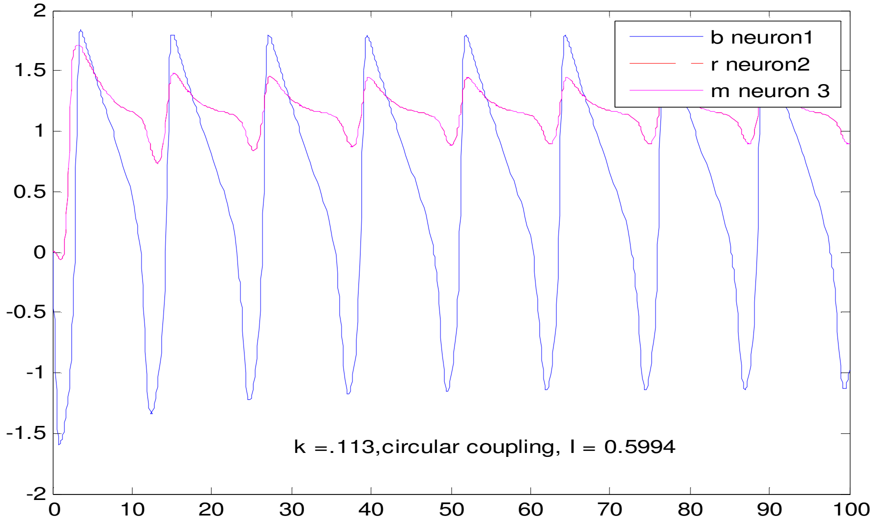

Fig 6: 3 neurons I > threshold, neuron 2 and 3 absolutely synchronized

Coupling constant K = 0.213

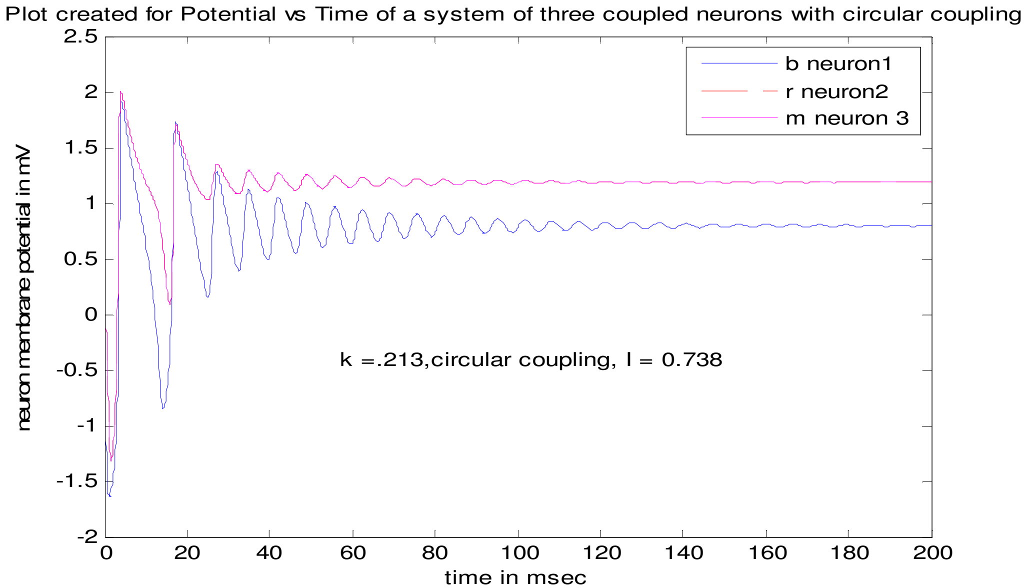

Fig 7: 3neurons, I< threshold

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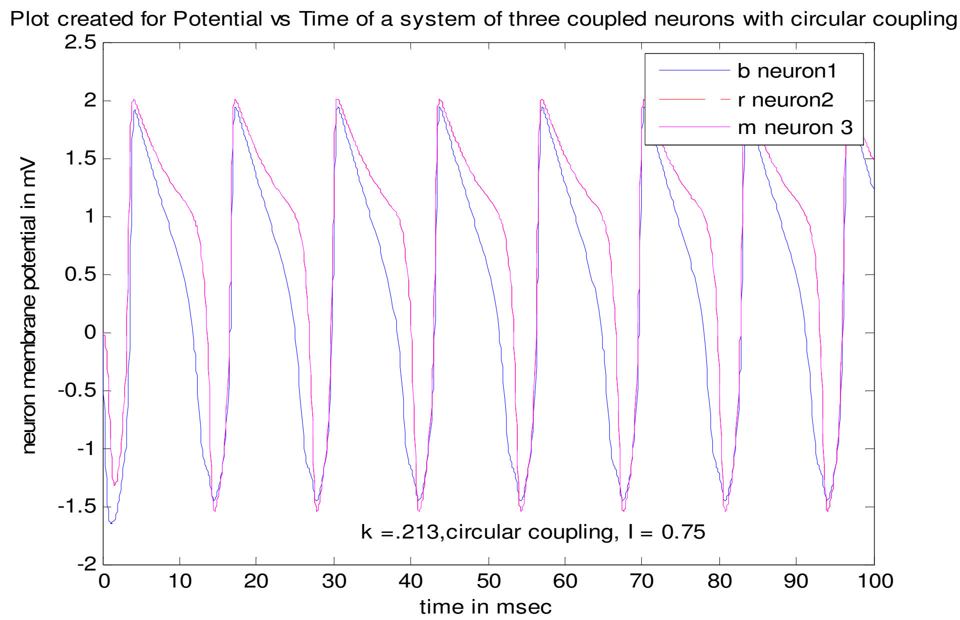

Fig 8: 3 neurons, I > threshold

Coupling Constant K =.313

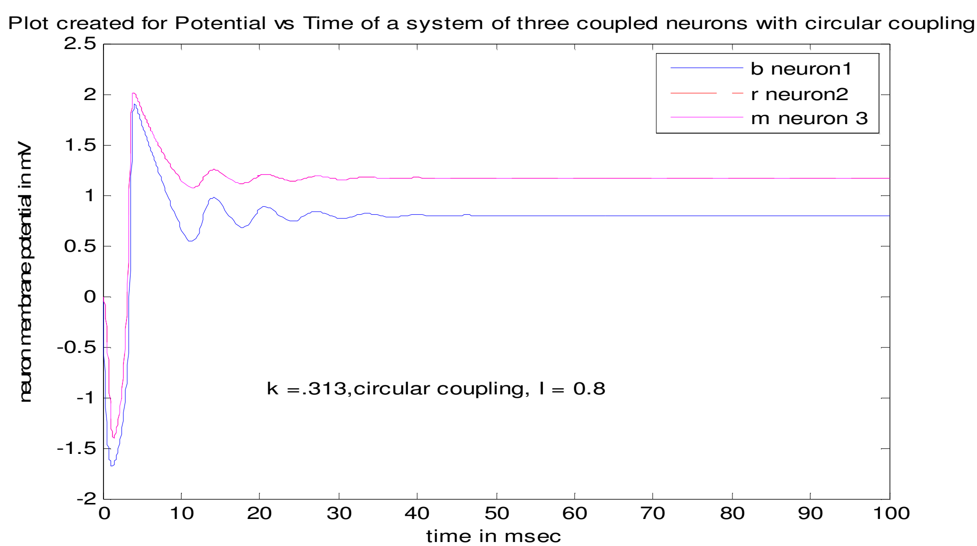

Fig 9:

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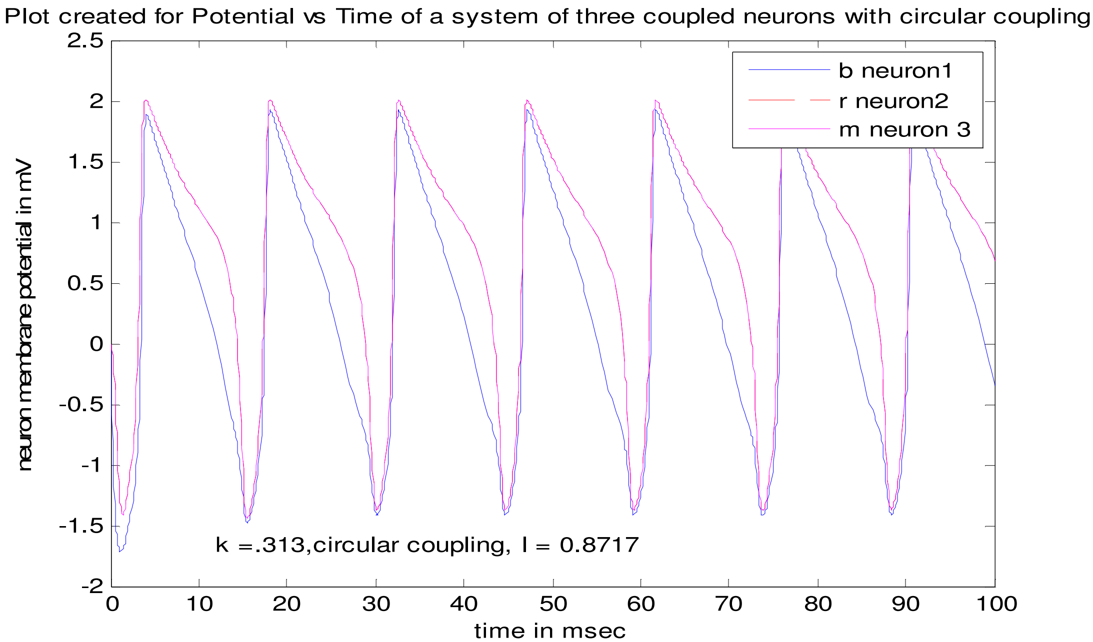

Fig 10: 3 neurons I > threshold

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ISBN: 978-93-83083-88-6