Hydromagnetic Thermal Instability of Solutes and Suspended Particles in a Fluid Layer in Porous Medium

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ABSTRACT

The paper examines the hydromagnetic thermal instability of solutes and suspended particles in a fluid layer in porous medium. The fluid is taken to be statically non-homogeneous confined between two horizontal boundaries and heated from below. We obtained the boundary conditions for the flow and found that the principle of exchange of stability (PES) is not valid for this system. The sufficient condition for stability of the system is also obtained in the paper. The frequency of oscillation at the marginal state and Rayleigh number are also examined. The characteristic equations for the non-oscillatory modes to be stable or unstable are also obtained.

Keywords: Thermosolutal fluid layer, porous medium, magnetic field, suspended particles

1. INTRODUCTION

The study of thermosolutal convection in porous medium in a heterogeneous fluid is of great importance. It may be important and may find applications in soil science, geophysics, biomechanics, and ground water hydrology and in many industrial problems such as oil recovery and in the chemical and nuclear industries. Recent studies of stellar atmosphere have shown the existence and importance of porosity in astrophysics. Chandrasekhar (1968) in his literary composition has given the comprehensive account of the investigations of various workers under different physical situations on the thermal stability of a homogeneous horizontal fluid layer in non-porous medium. Veronis (1965) and Nield (1967) have studied the thermosolutal convection under various assumptions and situations. The problem of thermal convection under porous medium was studied by Horton and Rogers (1945) and Lapwood (1948) and this work was extended by several workers such as Elder (1967), Nield (1968).

Gupta *et. al.* (1985) analyzed the thermohaline convection in horizontal fluid layer of Veronis type with a uniform vertical rotation and magnetic field between two rigid boundaries and showed that the complex growth rate ρ of an oscillatory perturbation is neutral or unstable with wave number

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$a^2 \geq \frac{\alpha}{2\sigma_1}$ must lie inside a semi-circle. They also showed that a similar result holds good for stern type thermohaline convection with a uniform rotation and magnetic field.

The effect of suspended particles in thermosolutal convection in porous medium was analyzed by Sharma and Rani (1987). They showed that for thermal Rayleigh number greater than or equal to solute Rayleigh number, principle of exchange of stabilities is valid and that the oscillatory modes may come into play if the thermal Rayleigh number is less than the solute Rayleigh number. Further, the effect of suspended particles is to destabilize the layer. It was also shown that the medium permeability and the stable solute gradient respectively have destabilizing and stabilizing effect on the system. Moreover, the rotation stabilizes a certain wave number range in thermosolutal convection in porous medium, which were unstable in the absence of rotation.

Sharma and Veena Kumari (1990) discussed the thermosolutal hydromagnetic instability in visco-elastic fluid layer heated from below. It was taken that the fluid had a statically stable solute gradient under a uniform magnetic field. They showed that both the magnetic field and solute gradient have stabilizing effect. Further, for stationary convection at the marginal state, the fluid behaves like a Newtonian fluid and that there was no contribution due to visco elastic character of the fluid.

Allah (2000) studied the stability of a stratified, incompressible fluid confined between two horizontal planes through porous medium in the presence of suspended particles, rotation and vertical oscillation. He showed that in the absence of porous media, it is found that the rotation and vertical oscillations have a stabilizing effect on the stability of a stratified fluid while suspended particles may be stabilizing or destabilizing depending on the parameter defining the direction of mass concentration of suspended particles. Further, he showed that the porosity has a destabilizing effect in the presence of rotation and vertical oscillations and in the absence of suspended particles.

Sharma and Aggarwal (2006) analyzed the effect of compressibility and suspended particles on thermal convection in Walters' B' Elastico-viscous fluid in hydromagnetics and found that compressibility and magnetic field has a stabilizing effect on the thermal stability. Rana and Kango (2011) have discussed the effect of suspended particles, rotation and magnetic field on the Thermosolutal instability in porous medium. They have derived the dispersion relation governing the effect of the solute concentration, suspended particles, rotation, magnetic field and medium permeability by applying normal mode analysis method. They found that the suspended particles have destabilizing effect whereas rotation and solute concentration have stabilizing effect on the system. The magnetic field and medium permeability have stabilizing/destabilizing effect on the system depending upon certain conditions.

In this paper, we have discussed the hydro magnetic thermosolutal instability of a fluid layer in the porous medium confined between free boundaries.

2. MATHEMATICAL FORMULATION OF THE PROBLEM:

Consider the stability of an incompressible, electrically conducting, viscous, density stratified fluid layer in a porous medium in the presence of a uniform magnetic field $(H,0,0)$ with solute concentration C. The fluid is taken to be statically non-homogeneous confined between two horizontal boundaries and heated from below. Let T_0 and $T_1 (T_1 < T_0)$ denote the uniform temperature of the lower and upper boundaries respectively. Then the governing equations of motion are

$$\rho\left[\frac{\partial V}{\partial t} + (V.\nabla)V\right] = -\nabla p - \frac{\mu}{\kappa}V + \mu\nabla^2 V + \rho X_i + \frac{\mu_l}{4\pi}[(\nabla\times H)\times H] \quad (1)$$

$$\nabla.V = 0 \quad (2)$$

$$\nabla.H = 0 \quad (3)$$

$$\frac{\partial T}{\partial t} + (V.\nabla)T = k_T\nabla^2 T \quad (4)$$

$$\frac{\partial C}{\partial t} + (V.\nabla)C = k_S\nabla^2 C \quad (5)$$

$$\frac{\partial H}{\partial t} + (V.\nabla)H = (H.\nabla)V + \eta\nabla^2 H \quad (6)$$

$$\rho = \rho_0\left[f(z) + \alpha(T_0 - T_1) + \alpha_1(C_0 - C_1)\right] \quad (7)$$

where $\rho, p, \mu_l, V(u,v,w), \mu, \kappa, k_T$ and C denote respectively the density, pressure, magnetic permeability, velocity component of the fluid, viscosity, medium permeability, thermal diffusivity

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of the fluid and solute concentration. k_S stands for coefficient of solute diffusion and ρ_0 is the fluid density at lower boundary at $z=0$. The whole system under force of gravity $X_i(0,0,-g)$ and $f(z)$ is a monotonic function of vertical coordinate z with $f(0)=1$.

Let the initial state of the system be characterized by the following solution for velocity of the fluid, temperature, concentration of the solute, density, and magnetic field, respectively as

$$V=(0,0,0) \tag{8}$$

$$T_1=T_0-\beta z, \qquad \beta=\frac{(T_0-T_1)}{d}>0 \tag{9}$$

$$C_1=C_0-\beta_1 z \tag{10}$$

$$\rho=\rho_0\left[f(z)+\alpha(T_0-T_1)+\alpha_1(C_0-C_1)\right] \tag{11}$$

$$H=(H,0,0) \tag{12}$$

where β represents the uniform adverse temperature gradient maintained between the plates and β_1 represents the solute concentration decreasing upward.

3. PERTURBATION STATE:

To analyze the stability of the fluid we perturbed the above basic state of the fluid layer, which is given by

$$V^*=(u,v,w) \tag{13}$$

$$T_1^*=T_1+\theta \tag{14}$$

$$C_1^*=C_1+S \tag{15}$$

$$\rho^*=\rho_0\left[f(z)+\frac{\partial\rho}{\rho_0}+\alpha(T_0-T_1-\theta)+\alpha_1(C_0-C_1-S)\right] \tag{16}$$

$$p^* = p + \delta p \tag{17}$$

$$H^* = \left(H + h_x, h_y, h_z\right) \tag{18}$$

where (u, v, w), $\theta, S, \delta\rho, \delta p$ and (h_x, h_y, h_z) are respectively the perturbation in the velocity of the fluid, temperature, solute concentration, density, pressure, and the magnetic field. Substituting these variables in the equation (1) to (7) and taking the perturbation variables to be arbitrarily small, we have the linearized perturbation equation as

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial \delta p}{\partial x} - \frac{\mu}{\kappa} u + \mu \nabla^2 u + \frac{\mu_l}{4\pi} H \frac{\partial h_x}{\partial x} \tag{19}$$

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial \delta p}{\partial y} - \frac{\mu}{\kappa} v + \mu \nabla^2 v + \frac{\mu_l}{4\pi} H \frac{\partial h_y}{\partial x} \tag{20}$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial \delta p}{\partial z} - \frac{\mu}{\kappa} w + \mu \nabla^2 w + \frac{\mu_l}{4\pi} H \frac{\partial h_z}{\partial x} - g\delta\rho + g\alpha\rho_0\theta - g\alpha_1\rho_0 S \tag{21}$$

$$\frac{\partial(\delta\rho)}{\partial t} = -\rho_0 w \frac{\partial f}{\partial z} \tag{22}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{23}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0 \tag{24}$$

$$\frac{\partial \theta}{\partial t} - \beta w = k_T \nabla^2 \theta \tag{25}$$

$$\frac{\partial S}{\partial t} - \beta_1 w = k_S \nabla^2 S \tag{26}$$

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$$\frac{\partial h_x}{\partial t} = H\frac{\partial u}{\partial x} + \eta\nabla^2 h_x \tag{27}$$

$$\frac{\partial h_y}{\partial t} = H\frac{\partial v}{\partial x} + \eta\nabla^2 h_y \tag{28}$$

$$\frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial x} + \eta\nabla^2 h_z \tag{29}$$

To discuss the stability of the fluid layer, we consider the perturbation to be of the form

$$f(x, y, z, t) = \sum f(z)\exp\left[i\left(k_x x + k_y y\right) + nt\right] \tag{30}$$

where $f(z)$ is some regular function of z representing the perturbation variable $f(x, y, z, t)$. In this $k^2 = k_x{}^2 + k_y{}^2$ is the wave number and n is the complex growth rate of the perturbation mode. We substitute this form of the perturbation variable in the perturbation equation (19) to (29). Thus we have

$$\rho_0 nu = -ik_x\delta p - \frac{\mu}{\kappa}u + \mu(D^2 - k^2)u + \frac{\mu_l}{4\pi}H_i k_x h_x \tag{31}$$

$$\rho_0 nv = -ik_y\delta p - \frac{\mu}{\kappa}v + \mu(D^2 - k^2)v + \frac{\mu_l}{4\pi}H_i k_x h_y \tag{32}$$

$$\rho_0 nw = -D\delta p - \frac{\mu}{\kappa}w + \mu(D^2 - k^2)w + \frac{\mu_l}{4\pi}H_i k_x h_z - g\delta\rho + g\alpha\rho_0\theta - g\alpha_1\rho_0 S \tag{33}$$

$$n\delta\rho = -\rho_0 w\frac{df}{dz} \tag{34}$$

$$k_x u + k_y v = iDw \tag{35}$$

$$k_x h_x + k_y h_y = iDh_z \tag{36}$$

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$$\left[n - k_T\left(D^2 - k^2\right)\right]\theta = \beta w \tag{37}$$

$$\left[n - k_S\left(D^2 - k^2\right)\right]S = \beta_1 w \tag{38}$$

$$nh_x = iHk_x u + \eta(D^2 - k^2)h_x \tag{39}$$

$$nh_y = iHk_x v + \eta(D^2 - k^2)h_y \tag{40}$$

$$nh_z = iHk_x w + \eta(D^2 - k^2)h_z \tag{41}$$

Multiplying the equation (31) by k_x and equation (32) by k_y and then adding, we get

$$\rho_0 nDw = -k^2\delta p - \frac{\mu}{\kappa}Dw + \mu(D^2 - k^2)Dw + \frac{\mu_l}{4\pi}H_i k_x Dh_z \tag{42}$$

Now multiplying the equation (33) by k^2 and differentiating the equation (42) and after subtracting, we get

$$n(D^2 - k^2)w \;= \upsilon(D^2 - k^2)^2 w + \frac{\upsilon}{k}(D^2 - k^2)w + \frac{\mu_l}{4\pi\rho_0}H_i k_x (D^2 - k^2)h_z$$

$$- \frac{gk^2\left(\frac{df}{dz}\right)}{n}w - gk^2\alpha\theta + gk^2\alpha_1 S \tag{43}$$

where $\upsilon = \dfrac{\mu}{\rho_0}$ represent the coefficient of viscosity.

Now normalizing the governing equations (43), (37), (38) and (41), thus we have

$$\left(D^2-a^2\right)\left[\left(D^2-a^2-\sigma-B\right)w+\frac{\mu_l H_i a_x d}{4\pi\rho_0\upsilon}h_z\right]-\frac{gd^4\left(\frac{df}{dz}\right)a^2}{\upsilon^2}w$$

$$-\frac{g\alpha a^2 d^2}{\upsilon}\theta-\frac{g\alpha_1 a^2 d^2}{\upsilon}S=0 \tag{44}$$

$$\left(D^2-a^2-p_r\sigma\right)\theta=-\left(\frac{\beta d^2}{k_T}\right)w \tag{45}$$

$$\left(\tau\left(D^2-a^2\right)-p_r\sigma\right)S=-\left(\frac{\beta_1 d^2}{k_T}\right)w \tag{46}$$

$$\left(D^2-a^2-p_1\sigma\right)h_z=\left(\frac{-iHa_x d}{\eta}\right)w \tag{47}$$

also, from equations (44) to (47) after using the dimensionless quantities

$$[\left(D^2-a^2\right)\left(D^2-a^2-p_r\sigma\right)\left(\tau(D^2-a^2)-p_r\sigma\right)$$

$$\left[\left(D^2-a^2-p_i\sigma\right)\left(D^2-a^2-\sigma-B\right)+Qa_x^2\right]+Ra^2\left(D^2-a^2-p_i\sigma\right)\left(\tau(D^2-a^2)-p_r\sigma\right)$$

$$-R_1a^2\left(D^2-a^2-p_i\sigma\right)\left(D^2-a^2-p_r\sigma\right)$$

$$-\frac{R_2a^2}{P_r}\left(D^2-a^2-p_i\sigma\right)\left(D^2-a^2-p_r\sigma\right)\left(\tau(D^2-a^2)-p_r\sigma\right)]\text{w=0} \tag{48}$$

The dimensionless quantities are

$D^*=dD$, $a=kd$, $\sigma=\dfrac{nd^2}{\upsilon}$, $p_r=\dfrac{\upsilon}{k_T}$, $p_i=\dfrac{\upsilon}{\eta}$,

$\tau=\dfrac{k_S}{k_T}$, $B=\dfrac{d^2}{\kappa}$, $\alpha=\dfrac{\mu_l H^2 d^2}{4\pi\rho_0\upsilon n}$, $R=\dfrac{g\alpha\beta d^4}{k_T\upsilon}$,

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$$R_1 = \frac{g\alpha_1\beta_1 d^4}{k_T\upsilon}, \qquad R_2 = \frac{gd^4\left(\frac{df}{dz}\right)}{k_T\upsilon} \tag{49}$$

In equations (44) to (48), star symbol on quantities defined in equation (49) have been dropped.

4. BOUNDARY CONDITIONS:

The boundaries are free and following Chandrasekhar (1961), the boundary conditions are

$$\left.\begin{array}{l} w = D^2 w = 0 \\ \theta = S = h_z = 0 \end{array}\right\} \quad \text{at } z = 0 \text{ and } z = d \tag{50}$$

5. PRINCIPLE OF EXCHANGE OF STABILITIES IS NOT VALID:

If possible, let us say that the principle of exchange of stabilities is valid, is characterized by $\sigma = 0$. Putting $\sigma = 0$ in equation (44) to (47), we find that the solution satisfying the boundary conditions (50) are

$$w = \theta = S = h_z = 0$$

This shows that the initial state solutions are not perturbed, which is a contradiction. Thus stationary marginal state cannot exist and the PES is invalid for this problem.

6. SUFFICIENT CONDITION FOR THE STABILITY OF THE SYSTEM:

In this section, we analyze the nature of perturbation modes, for this we will solve the eigen value problem consisting of equation (44) to (47) together with the boundary conditions (50) and take the solution of the form

$w = ASinn\pi z$, where A is a constant.

we take the smallest value of n, that is n=1 and take the solution as

$$w = ASin\pi z \tag{51}$$

Substitute this solution of w in equation (44) and (47), we get after eliminating h_z,

$$\left[\left(\pi^2+a^2\right)\left(\pi^2+a^2+p_1\sigma\right)\left(\pi^2+a^2+B+\sigma\right)+Qa_x^2\left(\pi^2+a^2\right)-\frac{R_2a^2}{p_r}\left(\pi^2+a^2+p_1\sigma\right)\right]w=$$

$$\left(\frac{R_1a^2k_T}{\beta_1d^2}\right)\left(D^2-a^2-p_1\sigma\right)S-\left(\frac{Ra^2k_T}{\beta d^2}\right)\left(D^2-a^2-p_1\sigma\right)\theta \tag{52}$$

From equation (45) and (46), we find the particular solution for θ and S. They are

$$\theta=\left(\frac{\beta d^2}{k_T}\right)\frac{1}{\left(\pi^2+a^2+p_r\sigma\right)}w \tag{53}$$

and

$$S=\left(\frac{\beta_1d^2}{k_T}\right)\frac{1}{\left[\tau(\pi^2+a^2)+p_r\sigma\right]}w \tag{54}$$

Now eliminating θ and S from equation (52) to (54), we get

$$A_0\left(A_0+p_1\sigma\right)\left(A_1+\sigma\right)+Qa_x^2A_0-\frac{R_2a^2}{\sigma p_r}\left(A_0+p_1\sigma\right)=\frac{Ra^2\left(A_0+p_1\sigma\right)}{\left(A_0+p_r\sigma\right)}-\frac{R_1a^2\left(A_0+p_1\sigma\right)}{\left(\tau A_0+p_r\sigma\right)}$$

(55)

where,

$$A_0=\pi^2+a^2$$
$$A_1=A_0+B$$

also equation (55) can be written in the form

$$\frac{Ra^2}{\left(A_0+p_r\sigma\right)}=A_0\left(A_1+\sigma\right)+\frac{Qa_x^2A_0}{\left(A_0+p_1\sigma\right)}-\frac{R_2a^2}{\sigma p_r}+\frac{R_1a^2}{\left(\tau A_0+p_r\sigma\right)} \tag{56}$$

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since σ is the complex growth rate of the perturbations and we can express $\sigma = \sigma_r + i\sigma_i$ where σ_r and σ_i are real and σ_i represents the oscillatory character of the perturbations. Substituting the value of σ in equation (56) and taking the real part of the equation, we have for non-oscillatory modes $\sigma_i = 0$,

$$\sigma_r \left\{ a^2 \left(\frac{R}{|A_0 + p_r\sigma|^2} - \frac{R_1\tau}{|\tau A_0 + p_r\sigma|^2} \right) p_r + \left(\frac{R_2 a^2}{|\sigma|^2 p_r} - \left(1 + \frac{Qa_x^2 p_1}{|A_0 + p_i\sigma|^2} \right) A_0 \right) \right\} +$$

$$\left\{ a^2 \left(\frac{R}{|A_0 + p_r\sigma|^2} - \frac{R_1\tau}{|\tau A_0 + p_r\sigma|^2} A_0 \right) - \left(A_0 A_1 + \frac{Qa_x^2}{|A_0 + p_i\sigma|^2} \right) \right\} = 0 \qquad (57)$$

It is clear from the above equation that if

$$R > \max.(R_1, \tau L), \qquad L = \frac{|A_0 + p_r\sigma|^2}{|\tau A_0 + p_r\sigma|^2}$$

and

$$R_2 > \frac{|\sigma|^2 p_r A_0}{a^2} \left[1 + \frac{Qa_x^2 p_1}{|A_0 + p_1\sigma|^2} \right]$$

This implies that coefficient of σ_r in equation (57) is definite positive while the last bracket term is positive or negative. Then the value of σ_r is either positive or negative. Therefore, non-oscillatory mode is either unstable or stable according as last bracket term is positive or negative.

7. MARGINAL STATE OF THE SYSTEM

In this system, we consider the marginal state of the system. For this take $\sigma_r = 0$ and we express $\sigma = i\sigma_1$, where σ_1 is real and represents the oscillatory character of the perturbation. Substituting this in the equation (56), we have

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$$\frac{Ra^2}{A_0 + ip_r\sigma_1} = A_0(A_1 + i\sigma_1) + \frac{Qa_x^2 A_0}{A_0 + ip_1\sigma_1} + \frac{iR_2 a^2}{\sigma_1 p_r} + \frac{R_1 a^2}{\tau A_0 + ip_r\sigma_1} \tag{58}$$

Separating the real and imaginary parts of the equation (58), we get

$$\frac{Ra^2 A_0}{A_0{}^2 + p_r{}^2\sigma_1{}^2} = A_0 A_1 + \frac{Qa_x^2 A_0}{A_0{}^2 + p_1{}^2\sigma_1{}^2} + \frac{R_1 a^2 \tau A_0}{\tau^2 A_0{}^2 + p_r{}^2\sigma_1{}^2} \tag{59}$$

and

$$\sigma_1^2\left[\frac{Qa_x^2 A_0 p_1}{A_0{}^2 + p_1{}^2\sigma_1{}^2} + \frac{R_1 a^2 p_r}{\tau^2 A_0{}^2 + p_r{}^2\sigma_1{}^2} - A_0 - \frac{Ra^2 p_r}{A_0{}^2 + p_r{}^2\sigma_1{}^2}\right] = \frac{R_2 a^2}{p_r} \tag{60}$$

After eliminating the Rayleigh number R from equation (59) and (60), we find the frequency of oscillation at the marginal state and the Rayleigh number is given by equation (59) and its minimum value is called the critical wave numbers a_C and frequency σ_C .

NATURE OF NON-OSCILLATORY MODES:

Let us say that non-oscillatory modes exist for which σ_1 is zero and $\sigma = \sigma_2$, σ_2 is real. Hence substituting $\sigma = \sigma_2$ in equation (56), we get

$$B_0\sigma^5 + B_1\sigma^4 + B_2\sigma^3 + B_3\sigma^2 + B_4\sigma + B_5 = 0 \tag{61}$$

where,

$$B_0 = p_1 A_0 p_r^3$$

$$B_1 = A_0 p_r^2 [A_0(p_1 + p_r) + (\tau A_0 + p_r A_1)p_1]$$

$$B_2 = A_0^2 p_r [p_r A_0 + (p_1 + p_2)(\tau A_0 + p_r A_1)] - Ra^2 p_r^2 p_1 + Qa_x^2 A_0 p_r^3 - R_2 a^2 p_r^2 p_1 + R_1 a^2 p_r^2 p_1$$

$$B_3 = p_r A_0^3 [(\tau A_0 + p_r A_1) + \tau A_1(p_1 + p_2) - pa^2 p_r^2 A_0(1+\tau) + Qa_x^2 A_0 p_r(A_0 p_r + \tau p_r) - R_2 a^2 p_r(p_1\tau + A_0(p_1 + p_2)) + R_1 a^2 p_r(p_1 + p_r)A_0]$$

$$B_4 = p_r A_0^4 \tau A_1 - Ra^2 p_r A_0^3 \tau A_1 + Qa_x^2 A_0^3 p_r \tau - R_2 a^2 \left[A_0^2 p_r + A_0^2 \tau(p_1 + p_r)\right] + R_1 a^2 p_r A_0(p_1 + p_r)$$

$$B_5 = -R_2 a^2 A_0^3 \tau$$

where,

$$A_0 = \pi^2 + a^2$$

and

$$A_1 = A_0 + B$$

Equation (61) is the characteristic equation which implies that

Since $R_2 > 0$ this implies that $B_5 < 0$ and product of all five roots is positive, therefore at least one root is positive, thus making the system unstable. Therefore, we conclude that non-oscillatory modes are unstable.

If $R_2 < 0$ this implies that $B_5 > 0$ and product of all five roots is negative, hence either all roots are negative or at least one root is positive thus both negative and positive roots are possible. Therefore, we conclude that if $R_2 < 0$ then non-oscillatory modes may be stable or unstable.

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