# Model Order Reduction by Pade Approximation and Improved Pole Clustering Technique

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***Abstract: The authors offered a mixed technique for reducing the order of the high order dynamic systems. A pole-clustering based method to derive a reduced order approximation for a stable single-input single- output (SISO) continuous time system is presented. In this method, the denominator polynomial of the reduced order model is obtained by improved pole- clustering approach and the numerator polynomial is obtained through Padé approximation technique. The reduced order model so obtained by improved clustering algorithm guaranteed the stability in the reduced model and also preserves the characteristics of the original system in the approximated one.***

***Keywords: Order reduction, Padé approximation, pole clustering, Dominant pole, Mean Square Error, Transfer function, IDM.***

## 1. INTRODUCTION

Model order reduction are often take interest in system modelling and design of high order systems. There are lots of methods proposed in the literature reflect the importance of producing a reduced order model for the system. It gives better considerate of the physical systems, reduced computational complexity, reduced hardware complexity, simplified controller design and cost effective solutions. The Padé approximation technique has been successfully applied to find the reduced system of higher order system, but sometimes this method has the drawback of resulting in unstable reduced order models of high order stable systems.

For the stable reduced order models, various methods based on the preservation of dominant poles have been proposed [2]. The reduced-order model retains the basic physical features (such as time constants) of the original system and the stability of the simplified model is guaranteed. In classical approach, the modes with the largest time constants i.e. slow modes or the poles nearest to imaginary axis are usually considered dominant. This approach is good, still with some demerits. Firstly slow modes may not be dominant, another in some cases the system may have all the poles arise in a small region in s-plane or modes with similar time constants, and lastly the for complex poles is not straightforward.

Various mixed methods based on the clustering of poles and Padé approximations are also proposed [4]. Methods describe the poles of the reduced system is use as the cluster centre of the pole clusters of the original system which obtained by Inverse distance measure (IDM) criteria. The choice of the clusters are either taken arbitrarily based on the order of reduction or it is the investigator. In this proposed method, the reduced order denominator polynomial has been obtained using an a dominant pole based pole-clustering approach for reduced order model. The method uses the improved clustering approach by deciding the value of the ratio of the residue to real parts of poles, taken in descending order and its corresponding reduced order model was obtained through a simple mathematical procedure. The model so obtained preserves the stability. The clustering method proposed in this paper differs from the existing pole clustering technique by considering the distance of system poles from the first pole in the group clustering process and it gives better approximation for order reduction.

## 2. PROBLEM STATEMENT

Consider an linear SISO time invariant system of $n^{th}$ order . Higher order transfer function be in the form

$$G(s)=\frac{q_0+q_1s+q_2s^2+\cdots+q_{m-1}s^{m-1}+q_ms^m}{p_0+p_1s+p_2s^2+\cdots+p_{n-1}s^{n-1}+p_ns^n} \quad (1)$$

Where m ≤ n

$$G(s)=\frac{N(s)}{D(s)}=\frac{\sum_{j=0}^{m}q_js^j}{\sum_{j=0}^{n}p_js^j} \quad (2)$$

Corresponding desired reduced order model of $r^{th}$ order should be given by

$$G_r(s)=\frac{a_0+a_1s+q_2s^2+\cdots+q_{l-1}s^{l-1}+a_ls^l}{y_0+y_1s+y_2s^2+\cdots+y_{r-1}y^{r-1}+y_rs^r} \quad (3)$$

Where l ≤ r

$$G_r(s)=\frac{N_r(s)}{D_r(s)}=\frac{\sum_{i=0}^{l}a_is^i}{\sum_{i=0}^{r}y_is^i} \quad (4)$$


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New denominator $D_r(s)$ for reduced order model is obtain through pole clustering technique

$$D_r(s) = (s - P_0)(s - P_1) \ldots\ldots (s - P_r) \quad (5)$$

New numerator $N_r(s)$ for reduced order model is obtain through Padé approximation technique as

$$\frac{N(s)}{D(s)} = \frac{N_r(s)}{D_r(s)}\ ;\ N_r(s) = D_r(s) * \frac{N(s)}{D(s)} \quad (6)$$

Here important characteristics of original system retain by obtained reduced order model through this mixed method of model order reduction.

## 3. DESCRIPTION OF PROPOSED METHOD

Computation of reduced order model consists two steps are as follows:

In first step find the denominator polynomial and in second step find numerator polynomial.

***Denominator polynomial computation***

The reduced denominator polynomial computed by improved pole clustering approach based on dominant pole and also by classical approach as shown below:

In dominant pole based pole-clustering approach Consider the given system in (1) can be shown as

$$G(s) = \sum_{i=1}^{n} \frac{R_i}{s-\lambda_i} \quad (7)$$

By the given equation corresponding to every pole $\lambda_i$ is the ratio of residue to pole intended as

Ratio of residue to poles = $\frac{|R_i|}{|Re(\lambda_i)|}$

The poles $\lambda_i$ arrange in descending value of ratio of residue to poles and then n-poles $\lambda_1\ \lambda_2\ \lambda_{3\ldots\ldots}\lambda_n$
arranged in group as most dominant pole for r-clusters such that the poles are equal or in maximum number of clusters. Number of cluster and cluster center one from each cluster is depend upon the order of the reduced system. The maximum number of poles in per cluster are not limited but for the computation of cluster center have none of repeated pole in same cluster center. In cluster formation first cluster have most dominant poles next cluster have next most dominant poles and so on.

***Proposed algorithm for cluster center:***

Let k number of poles available in one cluster group as: $P_1$, $P_2$, $P_3$, ........ $P_k$ cluster center obtained by inverse distance measure. The arrangement of poles are as follows $|P_1| < |P_2| < |P_3| \ldots\ldots < |P_k|$. Cluster center for the reduced order system is obtained through the following procedure. The procedure described in first step is similar to the case of the method proposed in [5] but the pole cluster calculated in the proposed method is based on the improved pole clustering approach in that particular cluster center.

Step 3.1 - Let the k number of poles available be arrange as $|P_1| < |P_2| < |P_3| \ldots\ldots < |P_k|$,

Step 3.2 - Set M = 1,

Step 3.3 - Calculate for the pole cluster as

$$C_M = \left[\left(-\frac{1}{|P_1|} + \sum_{i=2}^{k} -1/|P_i - P_1|\right) \div k\right]^{-1},$$

Step 3.4 - Check if M = k, then the final cluster center is $C_C = C_M$ and terminate the process else go for next step.

Step 3.5 - Again set M = M+1,

Step 3.6 - The improved cluster center from

$$C_M = -\sqrt{P_1 * C_{M-1}}$$

Step 3.7 - Check M = k . If no then go to step 3.5 otherwise for next step,

Step 3.8 - The final cluster center is $C_C = C_M$ .

Here if the system to be reduced have pole lie on imaginary axis that should be retain as the cluster center as a single pole and other remaining are clustered in another clusters based on algorithm. The pole cluster centres computed by the proposed method are more dominant than the pole cluster centres as obtained from the method proposed in[5]. While calculating the r[th] order system to be obtained, cluster center are obtained. And finding the cluster center values, we have the following three cases as

Case(1) - If all denominator poles or cluster center obtained are real. The denominator polynomial of reduced order in the form as:

$$D_r(s) = (s + C_{c1})(s + C_{c2}) \ldots.\ (s + C_{cr}) \quad (8)$$

The improved cluster values given as $C_{c1}C_{c2} \ldots\ldots C_{cr}$ and r is the order of reduced system.Or we can represent it as given in equation(5).

Case(2) - If all poles or cluster center obtained are complex. The denominator polynomial of reduced order in the form as:

Let k complex conjugate poles in single cluster group be $(P_1 \pm jQ_1, P_2 \pm jQ_2, \ldots \ldots P_k \pm jQ_k)$. Where $|P_1| < |P_2| < |P_3| \ldots \ldots < |P_k|$ are the obtain through same algorithm is proposed above, it follows separately for real and complex poles. Then the improved cluster center is as

$$\beta_i = P_i \pm jQ_i$$

Corresponding reduced order polynomial is

$$D_r(s) = (s + |\beta_1|)(s + |\beta_2|) \ldots (s + |\beta_i|) \quad (9)$$

Case(3) - If some cluster center are real and remaining are in the complex form.Then applying the algorithm separately for real part and then for complex terms. To get cluster center for reduced system combine both of them to find denominator polynomial.

Classic dominance based pole clustering approach:
In classic dominance based approach pole cluster formation is made by most dominant pole first, and most dominant pole is decided through the modes with the largest time constants or nearest pole to the origin. And further cluster center is obtained through the same algorithm proposed.

***Denominator polynomial computation***

Denominator polynomial is find by Pade approximation in it equate the original higher order system transfer function with generated reduced system transfer function. The denominator polynomial of reduced system obtained from first step is use here to get the unknown coefficient of reduced system. As given

$$\frac{N(s)}{D(s)} = \frac{N_r(s)}{D_r(s)}$$

$$\frac{q_0+q_1s+q_2s^2+\cdots+q_{m-1}s^{m-1}+q_ms^m}{p_0+p_1s+p_2s^2+\cdots+p_{n-1}s^{n-1}+p_ns^n} = \frac{a_0+a_1s+q_2s^2+\cdots+q_{l-1}s^{l-1}+a_ls^l}{y_0+y_1s+y_2s^2+\cdots+y_{r-1}y^{r-1}+y_rs^r} \quad 10)$$

By cross multiplying the equation and comparing the coefficient of same power of 's'and we get the new numerator coefficient as

$$q_0y_0 = p_0a_0$$

$$q_0y_1 + q_1y_0 = p_0a_1 + p_1a_0$$

$$q_0y_2 + q_1y_1 + q_2y_0 = p_0a_2 + p_1a_1 + p_2a_0$$

$$q_my_r = p_na_l$$

By solving it the unknown coefficient $a_0, a_1, a_3, \ldots\ldots a_l$ can get easily. The numerator polynomial of reduced order system is shown in form of

$$N_r(s) = y_0 + y_1s + y_2s^2 + \ldots + y_{r-1}s^{r-1} + y_rs^r \quad (11)$$

## 4. MEAN SQUARE ERROR(MSE)

For further effectiveness of the proposed approach MSE is determine. The MSE is as

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_0(t_i) - y_r(t_i))^2$$

Here $y_0(t)$ is the step response of original higher order system and $y_r(t_i)$ is step response of reduced order model.

**Numerical example**

Let us consider the 8th order system given in [8] described by the transfer function as:

$$G(s) = \frac{N(s)}{D(s)} = \frac{40320+185760s+222088s^2+122664s^3+}{40320+109584s+118124s^2+67284s^3+}\ldots$$

$$\ldots\frac{+36380s^4+5982s^5+514s^6+18s^7}{+22449s^4+4536s^5+546s^6+36s^7+s^8}$$

It can be represent in the form of (7) as

$$G(s) = \frac{-3.0004}{s+1} + \frac{4.0444}{s+2} + \frac{-6.6750}{s+3} + \frac{11.5556}{s+4} + \frac{-3.6807}{s+5} + \frac{-1.2}{s+6} + \frac{-7.3306}{s+7} + \frac{9.6254}{s+8}$$

Poles of the system are
-1, -2, -3, -4, -5, -6, -7, -8

Dominant pole based pole clustering Approach

By the dominance method of ratio to residue the poles are in order as given below
-1, -4, -3, -2, -8, -7, -5, -6

To reduce the model into third order by DPPCA there are three cluster are formed as
Cluster (1) – Poles (-1, -4, -3)
Cluster (2) – Poles (-2, -8, -7)
Cluster (3) – Poles (-5, -6)

Cluster center calculated by improved pole clustering method as in algorithm

Reduced order model of system is as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{10.75\,s^2 + 26.63\,s + 7.49}{s^3 + 6.312\,s^2 + 12.48\,s + 7.49}$$

Graphical representation is shown in fig. 1


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Now by classical approach
In this case poles of same system are sorted by classical approach as in form
-1, -2, -3, -4, -5, -6, -7, -8

Cluster formation based on dominance criteria a
Cluster (1) – Poles (-1, -2, -3)
Cluster (2) – Poles (-4, -5, -6)
Cluster (3) – Poles (-8, -7)

Cluster center calculated as same algorithm as proposed.
Reduced order model of system is

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{13.64\, s^2 + 40.1\, s + 11.45}{s^3 + 8.019\, s^2 + 18.47\, s + 11.45}$$

Graphical representation of this system is shown in fig. 2

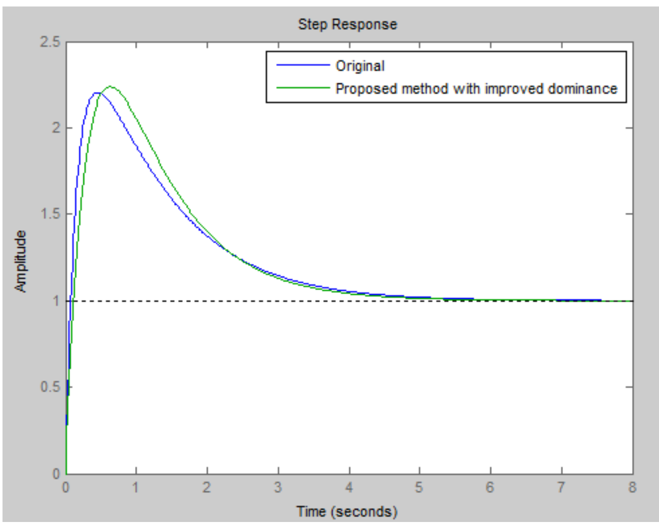


**Fig. 1. Step response comparison of original and reduced system with improved dominance**

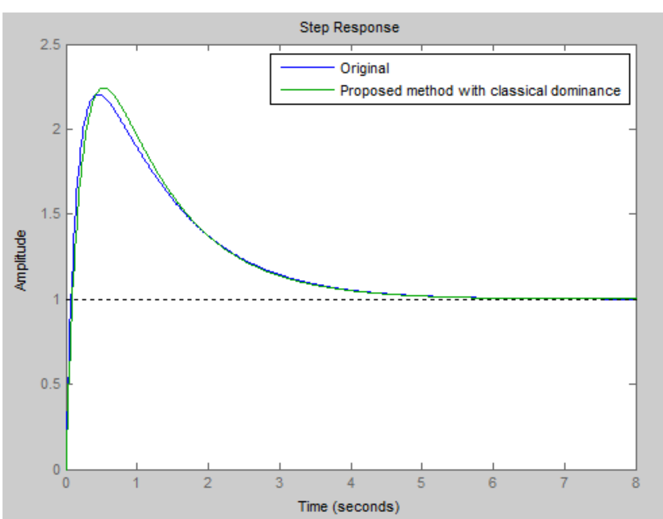


**Fig. 2. Step response comparison of original and reduced system with classical dominance**

**TABLE 1: MSE VALUE OF STEP RESPONSES**

| S. No. | Approach | MSE |
|---|---|---|
| 1. | DPPCA | 0.0071 |
| 2. | Classical dominance | 0.0015 |

**TABLE 2: COMPARISION OF PARAMETERS OF RESPONSES**

| Approach | Rise Time | Settling Time | Peak Time | Peak |
|---|---|---|---|---|
| Original system | 0.064 | 4.820 | 0.449 | 2.203 |
| DPPCA | 0.103 | 4.463 | 0.636 | 2.237 |
| Classical dominance | 0.082 | 4.711 | 0.531 | 2.242 |

## 5. CONCLUSION

In the proposed mixed method of reducing the model of higher order using improved pole clustering approach applying with the DPPCA and classical dominance approaches. Here denominator polynomial is derived by pole clustering and numerator polynomial through Pade approximation to get the reduced order model.

The comparison of MSE shows that proposed improved pole clustering approach with classical dominance perform better over DPPCA. The stability and characteristics of original system are retain in reduced order model. This method can be more improve easily by further modification as it is let go to research.

## REFERENCES

[1] R. Komarasamy, N. Albhonso & G. Gurusamy "Order reduction of linear systems with an improved pole clustering" *Journal of Vibration and Control*18(12) 1876–1885, 2011.

[2] Y. Shamash, "Linear system reduction using Padé approximations to allow retention of dominant poles, " *Int. J. control*, vol. 21, no. 2, pp 257-272, 1975.

[3] Y. Shamash, "Viability of methods for generating stable reduced order models, " *IEEE trans. Automatic control, vol*. 26, no. 6, pp 1285-1286, 1981.

[4] Vishwakarma C.B., Prasad R., "System reduction using modified pole clustering and Padé approximation, " *xxxii national systems conference*, NSC-2008 .

[5] Vishwakarma CB & Prasad R (2009). "MIMO system reduction using modified pole clustering and genetic algorithm", *Modelling and Simulation in Engineering.* DOI:10.1155/ 2009/540895.

[6] A.kumar D. Chandra "Improved Padé-pole clustering approximant" *International Conference on Computer Science and Electronics Engineering.*

[7] W. T. Beyene, “Pole-clustering and rational-interpolation techniques for simplifying distributed systems, ” *IEEE Trans. on Circuits and Systems-I: Fundamental Theory and Applications*, vol. 46, no. 12, pp.1468-1472, 1999.

[8] M. F. Hutton, & B. Friedland, “Routh approximation for reducing order of linear time invariant system”, *IEEE Trans. Auto. Cont*., AC-20, pp. 329-337, 1975.

[9] B. Vishwakarma, & R. Prasad, “Clustering method for reducing order of linear system using Padé approximation, ” *IETE Journal of Research*, vol. 54, no. 5, pp. 326-330, 2008.

[10] V. Singh, D Chandra, & H. Kar, "Improved Routh-Padé approximants: a computer-aided approach, " *IEEE, Trans. on automat. Contr*., Vol. 49, pp. 292-296, 2004.

[11] Parmar G, Prasad R and Mukherjee S (2007). “Order reduction of linear dynamic systems using stability equation method and GA”. *International Journal of Computer, Information, and Systems Science, and Engineering* 1(1): 26–32.

[12] Gupta DK, Bhagat SK & Tewari JP (2002). “A mixed method for the simplification of linear dynamic systems”. *In Proceedings of International Conference on Computer Applications in Electrical Engineering, Roorkee, India*, Feb. 21–23, pp. 455–459.


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