

Bivariate Interpolation Methods Conceptualized for Effective Optimization

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Abstract: Optimization is a must for every design so as to increase the performance of every mechanical system. Taking the example of helical compression spring, A new optimisation technique is found out by using an interpolation method or surface fit method. From many variables, the problem is reduced to 3 variables (x, y,z) and the variation of x with respect to z keeping y constant and variation of the y with respect to z keeping x constant is found out. Then the relation $z=f(x,y)$ is found out using Bivariate interpolation methods. The surface plot is obtained and for specific value of z, the 3D surface plot is chopped off, the resultant surface is the optimum surface which satisfies all the design considerations.

Taking an example of spring, keeping the parameters like free length, no of coils constant, the variation of spring index and diameter of wire is calculated monitoring deflection and factor of safety. A surface is fit to obtain to relate FOS to diameter of coil and spring index, for a particular value of say FOS=1, the obtained surface is chopped off, the surface obtained after chopping off is the optimum curve surface for which all conditions of design is satisfied.

Thus this method can be effectively used for the optimization of any 2 variable parameters with respect to the third.

1. INTRODUCTION

In the current engineering, optimization plays an important role in order to make the system more wiser and efficient. Optimization of the system makes the design simpler and use the resources to utilize to its best and obtain highly accurate results. Whenever optimization of mechanical system is concerned the purpose is to increase the efficiency and to decrease the weight of the system. In order to get the results of in between nodes or element, we usually go for interpolation methods. If the variable is a function of 2 variables, interpolation can be done using bivariate interpolation methods. Lagrangian bivariate method is very appropriate method to use in order to use even in case of unequal samples.

2. CONCEPT

The problem is reduced to 3 variables x,y,z. Statistical data which defines the relation between x, y and z are taken. Using

Any of the bivariate interpolation method, The Relation $z=f(x,y)$ is obtained, The surface is plot using the obtained equation, The surface plot is chopped off for the required value of z, so that the surface obtained using the chopping is the curve which satisfies the considered design parameters and the curve is the optimum curve.

3. APPLICATION TO SPRING

3.1 Problem Statement

A plate is constrained at one end, other end is free to move which is to be held inclined by the spring force. When the vehicle passes on it, the inclined plate gets deflected compressing the spring and plate gradually returns to its initial position once the vehicle passes over it. The design of the spring is to be in such a way that the spring should deflect completely when a small vehicle, like bike passes on it and should not be stressed more even if a heavy loaded truck of 40,000 kg passes on it. The solve this problem we have come with a solution of assembling the spring 100 mm below the ground with the free length of 200 mm without action of any load. But while designing the spring, we have to make sure that the spring is pre-stressed with the Plate of (3200 mm x 200 mm x 8 mm) which weighs 27 kg. Now the spring is kept under the plate and other end is buried 100 mm deep the road and fixed to ground

2.2 Defined boundary condition

- A pre-stressed plate of 27 kg.
- Max kg- force of vehicle that should pass is 200 kg.
- Max deflection for max load is 76 mm
- Free length = 200 mm.

2.3 Property of spring

The special properties of spring which made us to choose this concept is as follows

1. Fatigue load is always pulsating in case of spring, but not reversed
2. Yield Strength varies with the diameter of the wire

3. Wahl's stress factor takes care of direct stress as well as bending stress.
4. Overload just closes the gap between coils (reduces pitch) without a dangerous increase in the deflection.

The above mentioned properties will take care of all the possible worst case condition that can occur in the Speed Breaker application. The second property will give relation between yield strength and wire diameter, hence optimization of the diameter will take care of material requirements. Fourth property will take care of the factor of overloading, if the spring is made sure that the compressed length is not equal to the solid length, there will not be any dangerous deflection in the spring, for the same reason the spring is kept 100 mm inside the ground.

4. OPTIMIZATION

First by varying the carbon percentage the properties of the spring is found out in order to relate material property with spring diameter.

Patented and cold drawn spring steel is taken and all four grades of the steel is analyzed.

3.1 Grade 1

The grade 1 Spring Steel have following properties:

- Carbon – 0.50% - 0.75 %
- Silicon – 0.15% - 0.35%
- Manganese – 1.00%
- Copper- 0.120 %

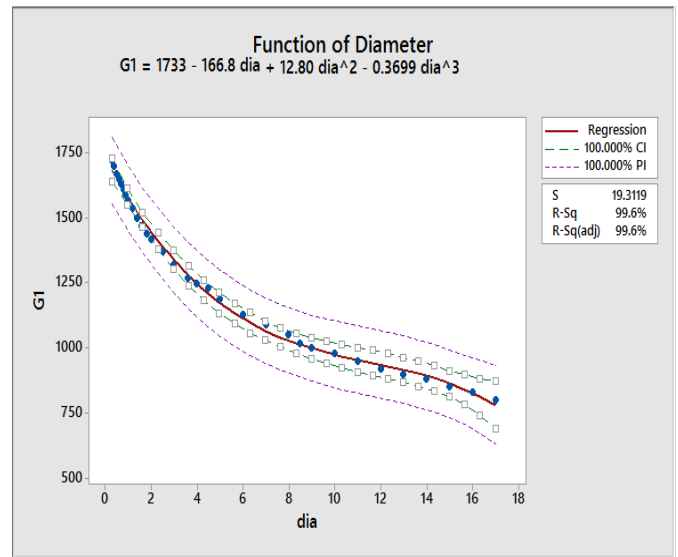
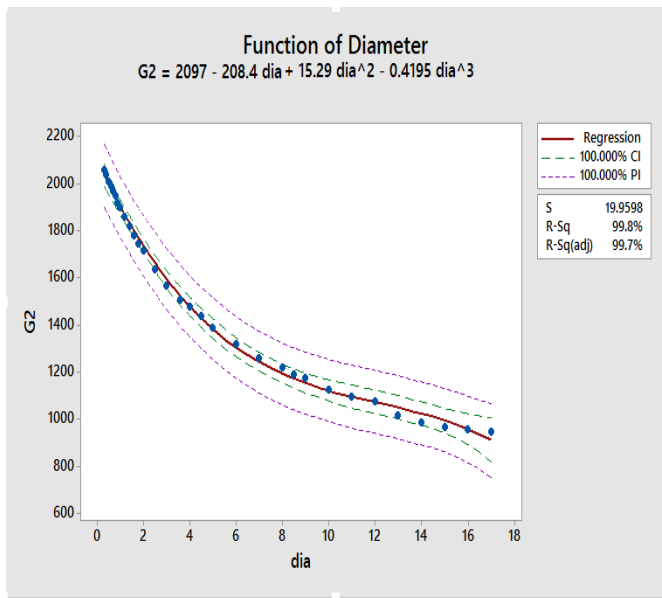


Fig-1. Grade 1 Spring Steel Strength function of diameter

Using Minitab statistical software, the relation between the tensile strength of the material with respect to the wire diameter is found out with 99.99% Confidence Interval and the R-sq value is found to be 99.6% which is much greater when compared to allowable percentage of 75%, hence this method can be used to for the further use.

The equation is found out to be $\sigma_{ut} = 1733 - 166.8d + 12.80d^2 - 0.3699d^3$

3.2 Grade 2 Steel

The grade 2 Spring Steel have following properties:

- Carbon – 0.60%-0.85%
- Silicon – 0.15%-0.35%
- Manganese – 0.80%
- Copper- 0.120%

From the Minitab software, the relation between ultimate tensile strength and diameter of wire is found out and equation is found to be

$$\sigma_{ut} = 2097 - 208.4d + 15.29d^2 - 0.4195d^3$$

Fig 2. Grade 2 Spring Steel Strength function of diameter

3.3 Grade 3 spring steel

The grade 3 spring steel have following properties

- Carbon – 0.75%-1.00%
- Silicon – 0.15%-0.35%
- Manganese – 0.80%
- Copper- 0.120%

From the Minitab software, the relation between ultimate tensile strength and diameter of wire and the obtained relation is used for the calculation of fatigue strength of the spring and relation found to be.

$$\sigma_{ut} = 2490 - 272.3d + 21.38d^2 - 0.6025d^3$$

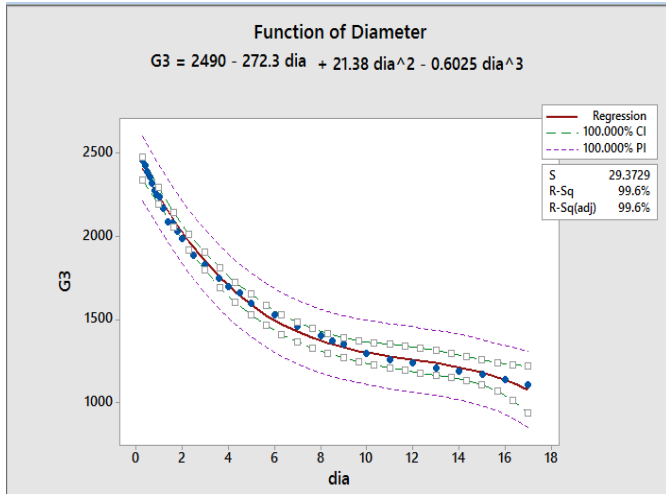


Fig-3. Grade 3 Spring Steel Strength function of diameter

3.4 Grade 4 spring steel

The grade 4 spring steel have following properties

- Carbon – 0.75%-1.10%
- Silicon – 0.15%-0.35%
- Manganese – 0.80%
- Copper- 0.120%

From the Minitab software, the relation between ultimate tensile strength and diameter of wire and the obtained relation is used for the calculation of fatigue strength of the spring and relation is found to be

$$\sigma_{ut} = 2699 - 301.9d + 25.10d^2 - 0.7343d^3$$

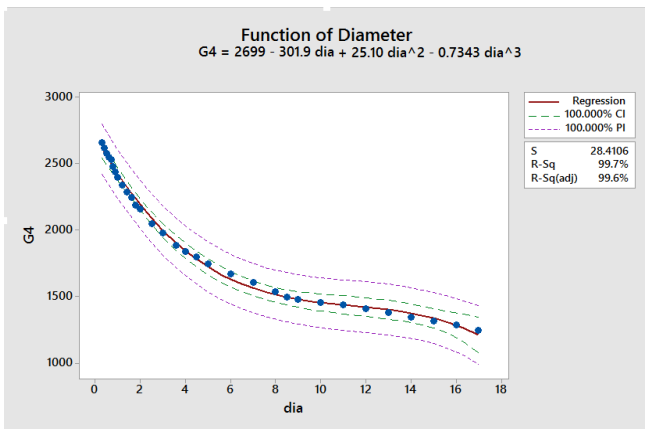


Fig-4. Grade 4 Spring Steel Strength function of diameter

As the grade 4 steel is stronger when compared to all other, the optimisation is done on this grade.

The Factor of safety is kept in z axis varying diameter of the wire and the spring in x and y axis respectively to obtain the relation.

The sampling points are collected, 9 to 16 samples are enough to get the exact results, but for more accuracy 20 samples are considered as shown below.

Table 1: Sampling points

d/c	6	8	10	12
6	0.55881	0.43961	0.896281	0.307733
8	0.92143	0.72488	0.59668	0.50677
10	1.38408	1.08847	0.896281	0.76121
12	1.94645	1.53135	1.26052	1.07057
14	2.566291	2.01888	1.66183	1.4114091

5. SURFACE PLOT

The bivariate interpolation of the above samples is carried out and equation of factor of safety as a function of spring index and diameter of wire.

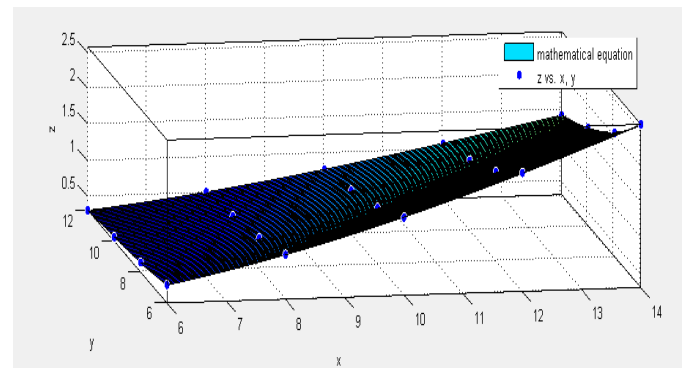


Fig 5, surface plot obtained in matlab

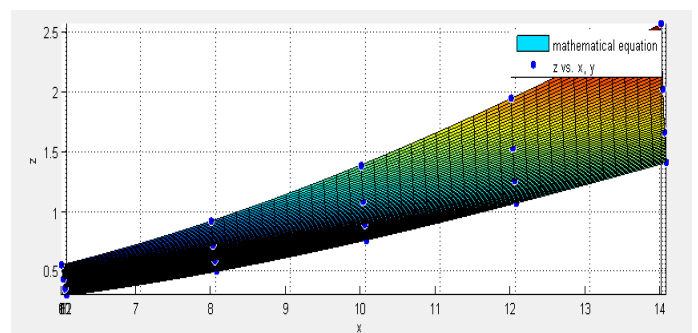


Fig 6: surface plot as the seen from xz plane

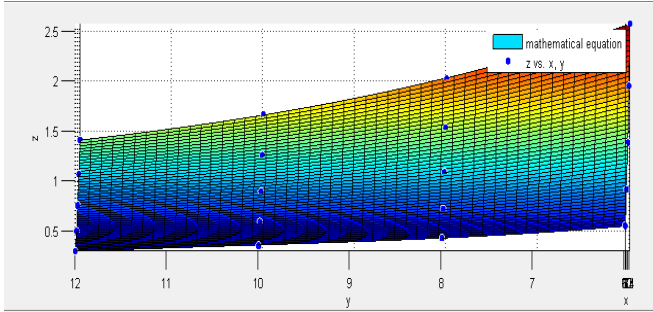


Fig 7: surface plot seen from yz plane.

The x (Diameter of wire) is varied only between the limits 6 mm to 15 mm in this but it is not restricted, whereas for the spring system to be stable the y (spring index) should be varied only between 6 to 12.

The above graph obtained has the following properties
 SSE: 0.0003092
 R-Square- 0.99998
 RSME- 0.005561

As the R-square value is greater than 0.75 with the confidence level of 99.98%, the plot can be used for the other applications.

The equation of the plot is

$$z = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2 + p30*x^3 + p21*x^2*y + p12*x*y^2 + p03*y^3$$

Where co-efficient are

- p00 = 0.3236
- p10 = 0.08935
- p01 = -0.09792
- p20 = 0.02552
- p11 = -0.03522
- p02 = 0.01963
- p30 = -0.000333
- p21 = -0.0008153
- p12 = 0.00182
- p03 = -0.001006

The above equation can be chopped off for the value of z=1 i.e factor of safety=1, the intersection of the surface z=1 and the surface above given is the optimum curve.

The Equation of the surface is given by

$$\text{Diameter of wire} = 8.645 - 0.5361C + 0.1043C^2$$

Within the limits diameter varies from 10 to 15 and spring constant 7 to 12.

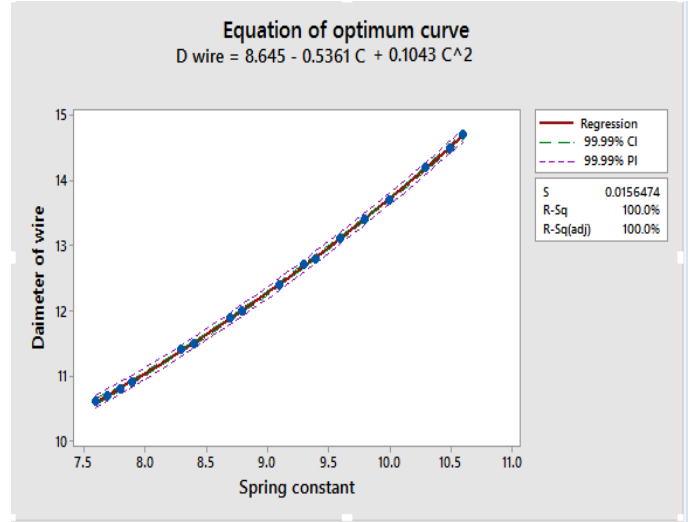


Fig 8. optimum graph

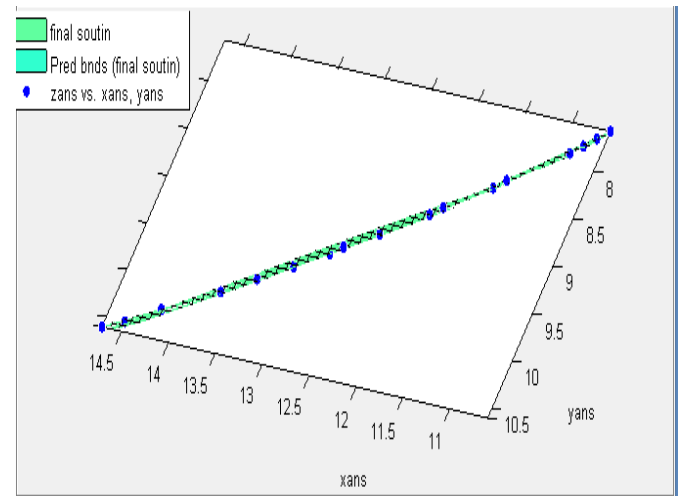


Fig 9: optimum curve in surface form

The value of z=1 and the variation of the x and y is as shown above.

The equation of the curve in surface form is found to be

$$1 = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2$$

Coefficients (with 99% confidence bounds):

- p00 = -0.7795
- p10 = 0.357
- p01 = -0.09745
- p20 = -0.01838
- p11 = 0.03398
- p02 = -0.0334

The optimum curve satisfies the design considerations, for the application of spring the mean (center) value is chosen with diameter of spring=12.5mm and the spring index = 8 which satisfies all the design parameters required.

6. APPLICATION

- Damping of Suspension system.
- Wall Thickness of space frame chassis.
- Thermal insulation of material.
- And many more.

7. CONCLUSION

In this paper we have tried to explain the concept with the help of the spring varying few parameters, But the same concept can be extended to all design problems which involves 3 variables.

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