A Study on Frequency Analysis for Puthimari Catchment by Gumbel Distribution Method

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Abstract: This paper presents results of a study carried in Assam, aimed at analyzing flood frequency of Puthimari catchment using Gumbel Distribution method. Design Rainfall estimates have been carried out on the Puthimari catchment employing flood frequency analysis methods for planning and for infrastructural design. This method consists of fitting a theoretical extreme-value probability distribution to the maximum annual rainfall data collected at a puthimari gauge station, thus enabling the hydrologist to estimate the peak Rainfall corresponding to a given design return period. Annual rainfall data of recent years (2001-2013) were collected. The estimated design rainfall helps for design of hydraulic structures like bridges, culverts, etc. The results of the investigation are analyzed and discussed and the useful conclusions are drawn. A frequency cure is also established, from where the dependable rainfalls can be calculated at various percentages.

Keywords: Gumbel distribution, frequency analysis, design rainfall, dependable rainfall

1. INTRODUCTION

Puthimari River, mainly, flows through Kamrup and Baksa district in Assam. It originates from the foothills of the Himalayan Ranges in Bhutan at an altitude of 3750m above M.S.L. It is known as Oontany River in Bhutan up to Indo-Bhutan border, Bornodi after crossing the border and '**Puthimari**' after crossing Nagrijuli Tea Estate. It is 112km long, of which 32km lies in Bhutan while the rest 80km lies in Assam. It has a catchment area of 1787 km² (712 km² in hills and 1075 km² in plains).

Origin	Bhutan (foothills of Himalayan ranges)		
Location	Kamrup & Baksa district of Assam		
Latitude &	26°10'N-27°18'N & 91°27'E-91°50'E		
Longitude			
Area of	1787 sq.km (712 sq.km in hills n 1075		
Catchment	sq.km in plains)		
Length of River	112km (32km in Bhutan & 80km in		
	Assam)		

Design rainfall is defined as the instantaneous peak annual rainfall adopted for the design of a hydraulic structure for the proper drainage of runoff and also for the economic and hydrological factors. It is a rainfall runoff that the project can sustain without any substantial damage, either to the objects which it protects or its own structures

2. FREQUENCY ANALYSIS

Designers of drainage works and drainage structures commonly use one of two methods to determine the design discharge. These are:

- Select a design discharge from a time series of measured or calculated discharges that show a large variation.
- Select a design rainfall from a time series of variable rainfalls and calculate the corresponding discharge via a rainfall-runoff transformation

Because high discharges and rainfalls are comparatively infrequent, the selection of the design discharge can be based on the frequency with which these high values will be exceeded. This frequency of exceedance, or the design frequency, is the risk that the designer is willing to accept. Of course, the smaller the risk, the more costly are the drainage works and structures, and the less often their full capacity will be reached. Accordingly, the design frequency should be realistic - neither too high nor too low. Frequency analysis is an aid in determining the design discharge and design rainfall. In addition, it can be used to calculate the frequency of other hydrologic (or even non-hydrologic) events.

2.1 Gumbel extreme value distribution method

Gumbel in 1941 was the first to consider that the annual flood peaks are extreme value of flood in each of the annual series of recorded flood or rainfall. Hence, floods should follow the extreme value distribution. Let $Q_1, Q_2, Q_3, ..., Q_n$ are the annual extreme values of flood in a particular river of catchment area. In Gumbel method, the exceedance probability P of a given flow Q_t having a return period of T years being equalled or exceeded is given by:

$$P = 1 - e^{-e^{-y}}$$

Where, y is called '**reduced variate**', e is the base of Naperian logarithm.

Since,
$$P = \frac{1}{T_r}$$

Therefore, $\frac{1}{T_r} = 1 - e^{-e^{-y}}$

When size of the sample is infinite, $y = a (Q_t - Q_f)$

Where,
$$a = \frac{1}{0.78\sigma}$$

Here, σ is standard deviation which is given by:

$$\sigma = \sqrt{\frac{\sum (Q - \bar{Q})^2}{N - 1}}$$

Where,

$$\bar{Q} = \frac{Q_1 + Q_2 + \dots + Q_n}{N}$$

And

$$Q_f = \bar{Q} - 0.45\sigma$$

Therefore, using the above equations,

 $Q_t = \bar{Q} + (0.78y - 0.45)\sigma$

However, if the flood records available are for limited period with limited samples as suggested by Chow,

$$Q_t = Q + k\sigma$$

The frequency factor k is determine by the equation

 $\mathsf{K}=((\mathsf{Yt}-\overline{\mathsf{Yn}})/\mathsf{Sn})$

Where, Sn = standard deviation and

Yt = reduced variate

3. ANALYSIS

Monthly rainfall data is available from the year 2001 to 2013 of puthimari catchment. Frequency analysis is carried out for the maximum annual rainfall data and a graph is plotted rainfall against return period. A frequency curve is also plotted from which 50% and 75% dependable rainfall is calculated. The maximum annual rainfall data along with the year, used in the frequency analysis is presented in the column (1) and (2) of

Table 1. The detail calculation and the value obtained in the calculation of Gumbel method is presented in Table 1. The computations of design rainfall for various return periods are calculated by using Gumbel method the values are presented in **Table 2**.

TABLE 1: Frequency analysis by Gumbel Method

Year (1)	Maximum Rainfall (mm) (2)	Descending Order (X) (3)
2001	345.8	604.2
2002	448.8	537.6
2003	441.9	511
2004	508.5	508.5
2005	352	448.8
2006	262.5	441.9
2007	511	388.6
2008	314.2	352
2009	316.4	345.8
2010	537.6	316.4
2011	388.6	314.2
2012	604.2	262.5
2013	254	254
	∑X=	5285.5

Rank (4)	Return Period (5)	Probability (6)	X^2 (7)
1	14	0.071428571	365057.64
2	7	0.142857143	289013.76
3	4.666666667	0.214285714	261121
4	3.5	0.285714286	258572.25
5	2.8	0.357142857	201421.44
6	2.333333333	0.428571429	195275.61
7	2	0.5	151009.96
8	1.75	0.571428571	123904
9	1.555555556	0.642857143	119577.64
10	1.4	0.714285714	100108.96
11	1.272727273	0.785714286	98721.64
12	1.166666667	0.857142857	68906.25
13	1.076923077	0.928571429	64516
		∑X^2=	2297206.15

X bar	∑X^2/N	S
(8)	(9)	(10)
406.5769231	176708.1654	106.7866

X bar = $\sum X / N$

 $S = \sqrt{\left(\left(\sum X^2/N\right) - \left(\sum X/N\right)^2\right)}$

The frequency factor k is determine by the equation $K = ((Yt - \overline{Yn}) / Sn)$ Where, Sn = standard deviation and Yt = reduced variate

 TABLE 2: Design rainfall for various return period of puthimari catchment

Return Period (1)	X bar (2)	S (3)	Yt (4)	(Yn) ⁻ (5)
5	406.5769	106.7866	1.500392995	0.507
10	406.5769	106.7866	2.250955556	0.507
15	406.5769	106.7866	2.674416611	0.507
20	406.5769	106.7866	2.970913185	0.507
25	406.5769	106.7866	3.199293342	0.507
50	406.5769	106.7866	3.902824486	0.507
75	406.5769	106.7866	4.31174361	0.507
100	406.5769	106.7866	4.601160867	0.507
1000	406.5769	106.7866	6.908682432	0.507

Standard deviation (6)	K (7)	KS (8)	Design Annual Rainfall (9)
0.9972	0.996182305	106.3789214	512.9558214
0.9972	1.748852343	186.7539956	593.3308956
0.9972	2.173502418	232.1009333	638.6778333
0.9972	2.470831513	263.8516965	670.4285965
0.9972	2.69985293	288.3081149	694.8850149
0.9972	3.405359493	363.646762	770.223662
0.9972	3.815426805	407.436456	814.013356
0.9972	4.105656706	438.4291204	845.0060204
0.9972	6.419657473	685.5333947	1092.110295

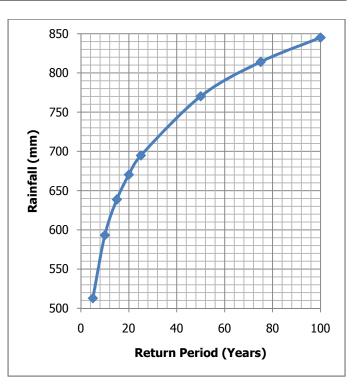


Fig. 1. Rainfall Vs return period of puthimari catchment

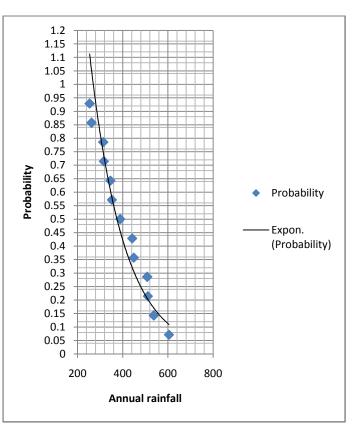


Fig. 2. Frequency curve or probability plot of puthimari catchment

4. CONCLUSION

Runoff estimation is of crucial importance and the major aspect of hydrologic design. Past rainfall data, its consistency and the validity is very important for accuracy in analysis. In this paper, design rainfall for different return periods have been computed by using Gumbel's distribution method on puthimari catchment. Rainfall data derived from the frequency analysis are needed by hydrologist and engineers involved in planning and design of any hydraulic structure in that place. From the frequency curve or probability plot in fig 2, the 75% and the 50% dependable rainfall are 312mm and 375 mm respectively. From the graph it state that , at puthimari catchment we can expect a rainfall of 312 mm or more in 75 years out of 100 years and a rainfall of 375mm or more in 50 years out of 100 years.

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