Signal Denoising and Multiresolution Analysis by Discrete Wavelet Transform

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ABSTRACT

One of the fields where wavelets have been successfully used is data analysis. Wavelet analysis is a relatively new and exciting method for solving difficult problems in mathematics physics, and engineering, with applications diverse as wave propagation, data compression, signal processing, image processing, pattern recognition, computer graphics,and other medical image technology. Wavelets allow complex information such as music, speech, images and patterns to be decomposed into elementary forms at different positions and scales and subsequently reconstructed with high precision. Wavelets have been found to be a powerful tool for removing noise from a variety of signals. They allow to analyze the noise level separately at each wavelet scale and to adapt the denoising algorithm accordingly. Wavelet thresholding methods for noise removal were first introduced by Donoho in 1993. The theoretical justifications and arguments in the favour are highly compelling. These methods do not require any particular assumptions about the nature of the signal, permits discontinuities and spatial variation in the signal, and exploits the spatially adaptive multiresolution of the wavelet transform.

The paper reviews and summarizes the use of wavelet transform for denoising signals contaminated with noise. This paper also discusses the diverse applications of the wavelet transform.

Keywords: wavelet, CWT.DWT.MRA, wavelet denoising, wavelet decomposition, thresholding

1. INTRODUCTION

Signal denoising is one of the most important and researched topics in the field of signal analysis. Signal denoising methods help to extract the useful and clear information from noisy environment and achieve the purpose of signal enhancement .Many approaches have been reported in the literature for the task of denoising, which can be roughly divided into two categories: denoising in the original signal domain (e.g., time or space) and denoising in the transform domain (e.g., Fourier or wavelet transform).

The development of wavelet transforms over the last two decades revolutionized modern signal and image processing, especially in the field of signal denoising. In 1982 Jean Morlet a French geophysicist, introduced the concept of a 'wavelet' . A wavelet is a wave-like oscillation with an amplitude that starts out at zero, increases, and then decreases back to zero. Unlike the sines used in Fourier transform for decomposition of a signal, wavelets are generally much more concentrated in time. They usually provide an analysis of the signal which is localized in both time and frequency, whereas Fourier transform is localized only in frequency.Many of the wavelet analysis applications can be attributed to signal denoising problems.

2. WAVELET TRANSFORM

Morlet first considered wavelets as a family of functions constructed from translations and dilations of a single function called the "mother wavelet" ψ(t), defined by

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), \qquad a,b \in \mathrm{R}\,,\ a \neq 0 \tag{1}$$

The parameter a is the scaling parameter or scale which measures the degree of compression. The parameter b is the translation parameter which determines the time location of the wavelet. If $|a| < 1$, then the wavelet in (1) is the compressed version (smaller support in time- domain) of the mother wavelet and corresponds mainly to higher frequencies. On the other hand, when $|a| > 1$, then $\psi_{a,b}(t)$ has a larger time-width than ψ(t) and corresponds to lower frequencies. Thus, wavelets have time-widths adapted to their frequencies. This is the main essence of the wavelets in signal processing and time-frequency signal analysis.[1]

3. TYPES OF WAVELET TRANSFORM

- Continuous time wavelet transform
- Discrete time wavelet transform

4. CONTINOUS TIME WAVE LET TRANSFORM

A continuous-time wavelet transform of *f(t)* is defined as:

$$CWT_{\psi} f(a,b) = W_f(b,a) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t)\psi^*(\frac{t-b}{a})\, dt \tag{2}$$

Here a, $b \in R$, $a \neq 0$ and they are dilating and translating coefficients, respectively. The asterisk denotes a complex conjugate. This multiplication of a is for energy normalization purposes so that the transformed signal will have the same energy at every scale. The wavelet transform decomposes the signal into different scales with different levels of resolution by dilating a single prototype function, the mother wavelet [2].

5. DISCRETE WAVELET TRANSFORM AND MULTIRESOLUTION ANALYSIS

One drawback of the CWT is that the representation of the signal is often redundant, since *a* and *b* are continuous over *R* (the real number). The original signal can be completely reconstructed by a sample version of $W(b,a)f$. Typically, we sample $W(b,a)f$ in dyadic grid, i.e.,

$$a = 2^{-m} \quad \text{and} \quad b = n\,2^{-m} \tag{3}$$

$m,n \in Z$, and Z is the set of positive integers discrete wavelet transform 0f the function f(t) is

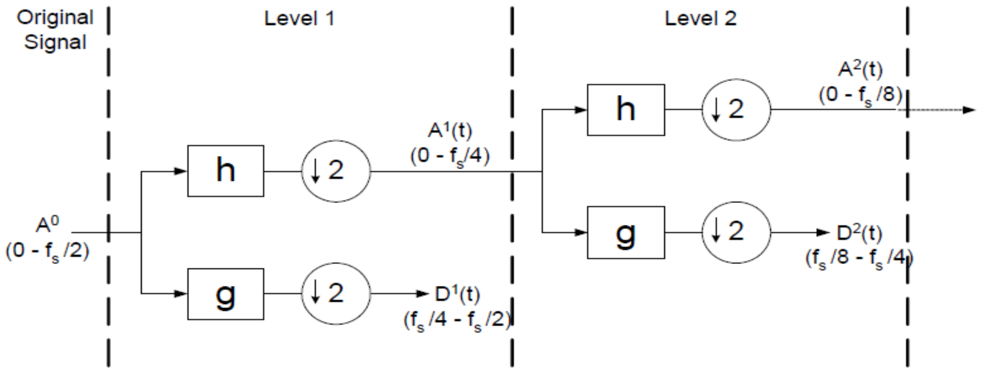
$$DWT_{\psi} f(m,n) = \int_{-\infty}^{\infty} f(t)\psi^{*}_{m,n}(t)\,dt \tag{4}$$

Where

$$\psi_{m,n}(t) = 2^{-m}\psi(2^{m}t - n) \tag{5}$$

is the dilated and translated version of the mother wavelet $\psi(t)$ .The family of dilated mother wavelets of selected *a* and *b* constitute an orthonormal basis of $L2(R)$ .. Due to the orthonormal properties, there is no information redundancy in the discrete wavelet transform. In addition, with this choice of *a* and *b*, there exists the multi resolution analysis (MRA) algorithm, which decompose a signal into scales with different time and frequency resolution. MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies.

The fundamental concept involved in MRA is to find the average features and the details of the signal via scalar products with scaling signals and wavelets. Wavelet transform proves to be a useful tool in analysis of non linear and non stationary signals. The algorithm of wavelet signal decomposition is illustrated in Fig 1. Reconstruction of the signal from the wavelet transform and post processing, the algorithm is shown in Fig 2. This multi-resolution analysis enables us to analyze the signal in different frequency bands; hence any transient in time domain or in frequency domain can be observed.[2]



Fig(1). Multi-resolution wavelet decomposition. h = low-pass decomposition filter; g = high-pass decomposition filter; 2 = down-sampling operation. A1(t), A2(t)are the approximated coefficient of the original signal at levels 1, 2 etc. D1(t), D2(t) are the detailed coefficient at levels 1,2

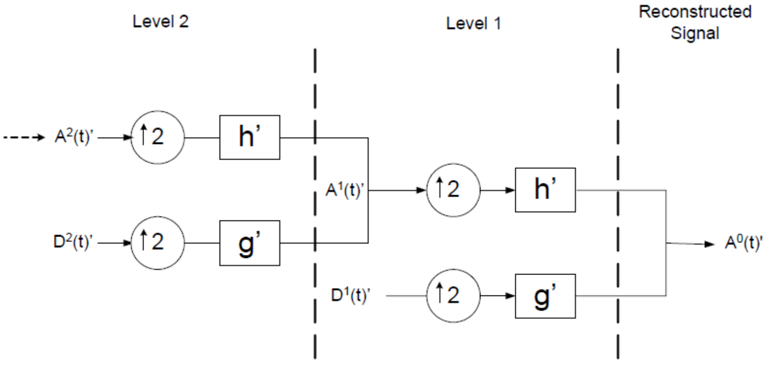


Fig (2) Multi-resolution wavelet reconstruction.

h' =low pass reconstruction filter g'=high-pass reconstruction filter; 2 = up-sampling operation. A1(t)' A2(t)' are the processed or non- processed approximated coefficient of the original signal at levels 1, 2 etc. D1(t)', D2(t)' are the processed or non processed detailed coefficient at levels 1,2.

The relation between the low-pass and high-pass filter and the scalar function $\psi(t)$ and the wavelet $\varphi(t)$ can be states as following

$$\phi(t) = \sum_k h[k]\phi[2t-k] \quad (6)$$

$$\psi(t) = \sum_k g[k]\phi[2t-k] \quad (7)$$

The relation between the low-pass filter and high-pass filter is not independent to each

Other, they are related by

$$g[L-1-n] = (-1)^n \cdot h[n] \quad (8)$$

Where g[n] is the high-pass, h[n] is the low-pass filter, L is the filter length (total number of points). Filters satisfying this condition are commonly used in signal processing, and they are known as the Quadrature Mirror Filters (QMF). The two filtering and down sampling operation can be

(9)

$$A^i[k] = \sum_n A^{i-1}(t) \cdot h[2k-n]$$

$$D^i[k] - \sum_n A^{i-1}(t) \cdot g[2k-n] \quad (10)$$

The reconstruction in this case is very easy since the half band filters form the orthonormal bases. The above procedure is followed in reverse order for there construction. The signals at every level are up sampled by two, passed through the synthesis filters g'[n], and h'[n] (high pass and low pass, respectively), and then added. The interesting point here is that the analysis and synthesis filters are identical to each other, except for a time reversal. Therefore, the reconstruction formula becomes (for each layer)[2]

$$A^i[k] = \sum_{k=-\infty}^{\infty} (D^{i+1}[k] \cdot g[-n+2k] + A^{i+1}[k] \cdot h[-n+2k]) \quad (11)$$

6. WAVELET BASED DENOISING

Principle of wavelet based denoising relies on the fact that signal magnitude dominates the magnitude of noise in a wavelet representation, so the wavelet coefficients can be set to zero if their magnitude are less than a predefined threshold.

Procedure can be as followed-

- Apply wavelet transform to the noisy signal to produce the noisy wavelet coefficients.
- Select appropriate threshold limit at each level and threshold method (hard or soft thresholding) to best remove the noises.

- Inverse wavelet transform of the thresholded wavelet coefficients to obtain a denoised signal.

7. (A). WAVELET DECOMPOSITION

It exists many ways to obtain the wavelet coefficients [3]. It has been compared two methods: the well-known classical structure of discrete wavelet transform with decimation and the same without the decimation (redundant or shift-invariant). On Figure 3 is shown block diagram of the second method where H(*z*) and Hr(*z*) are decomposition and reconstruction high pass filters. The G(*z*) and Gr(*z*) are low pass filters. The $d(\cdot,\cdot)$ are decomposition coefficients and $a(\cdot,\cdot)$ are approximation coefficients. For determination the better method to denoising were mentioned methods put to experiment. Input was a signal (*Doppler* generated with command wnoise with an additive noise[3]

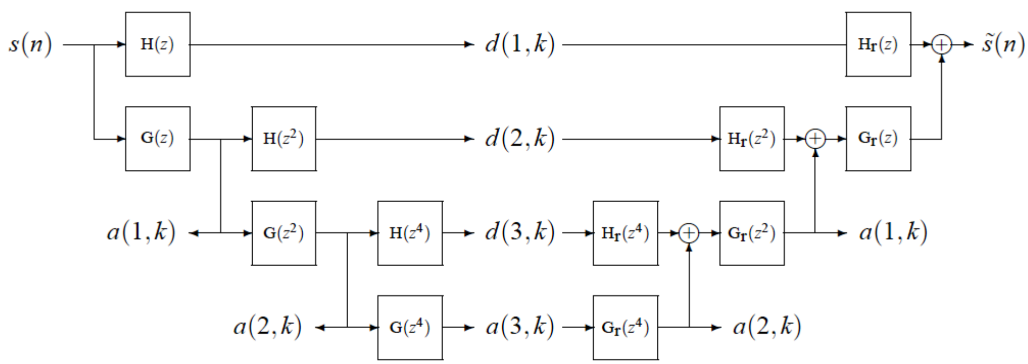


Figure (3): Shift invariant decomposition and reconstruction method

Signal to noise ratio (SNR) was measured =10 db by following relation

$$\mathrm{SNR}_{\mathrm{dB}} = 10\log\frac{\sum_{n=0}^{N-1} s(n)^2}{\sum_{n=0}^{N-1} v(n)^2} \approx 10\log\frac{\sum_{n=0}^{N-1} s(n)^2}{\sum_{n=0}^{N-1} (s(n)-\tilde{s}(n))^2} \qquad (12)$$

where $s(n)$ is a clean signal (without noise), $v(n)$ is a noise and ˜ $s(n)$ is an estimation of $s(n)$.This signal was decomposed of both structures to 6 levels. On Figure 3is demonstrated decomposition of the signal to 3levels.For decomposition were used different decomposition and reconstruction bank of filters eg. Daubechies(db5,db6,db) and biorthogonal (bior2.8, bior3.7, bior3.9 and bior6.8.In each level was looked for optimum size of threshold to get maximum SNR or minimum of Mean Square Error (MSE)[3]

$$\mathrm{MSE} = \mathbb{E}\left\{(s(n)-\tilde{s}(n))^2\right\} \approx \frac{1}{N}\sum_{n=0}^{N-1}(s(n)-\tilde{s}(n))^2 \qquad (13)$$

For minimize function MSE and determination responsible sizes of the thresholds was used Matlab generated function fminsearch. Results has been organized to the table

Table 1 Comparison of two decomposition method

	Shift-invariant		Transf. with decimation	
N	Filter	SNR [dB]	Filter	SNR [dB]
18	db5	18.4	db5	16.8
22	db6	18.6	db6	17.3
26	db7	18.7	db7	16.2
18	bior2.8	17.2	bior2.8	16.3
18	bior3.7	17.8	bior3.7	15.9
22	bior3.9	17.7	bior3.9	16.2
26	bior6.8	19.2	bior6.8	17.8

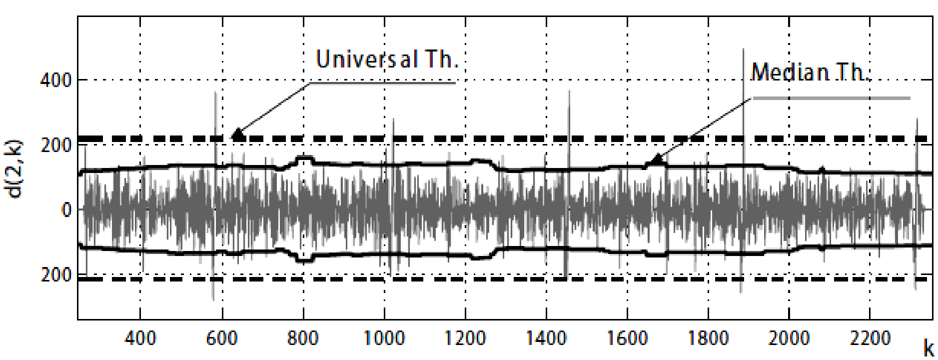
It can be seen that shift-invariant method of decomposition give better results in issue of Denoising [3]

8. THRESHOLDING THE WAVELET COEFFICIENT

Wavelet coefficients are modifying by Thresholding. In case of denoising is used soft thresholding it means that every coefficient which is higher then threshold is decreased by size of threshold. These coefficients which are under the threshold are deleted. Size of the threshold can be determined as universal threshold by the relation

$$\lambda = \sigma\sqrt{2\log N}, \tag{14}$$

Universal and Median Threshold



Figure(4) universal and adaptive threshold

Where λ is standard deviation and N is length of signal in each frequency level. This threshold is too high. Better is to use adaptive threshold obtained by order statistics filter. On the Figure [4] is shown second level wavelet coefficients, computed universal and adaptive threshold SNR=21.7dB, for FIR filtering SNR=13.2dB when SNR of the input signal was SNR=12.4dB.[3]

9. WAVELET AND ITS RECENT APPLICATIONS

Owing to its impressive performance Wavelet transform has been used to address several problems in the field of science and engineering. To name a few- Smoothing and image denoising, Fingerprint verification, DNA analysis, protein analysis, Blood-pressure, heart-rate and ECG analyses, Finance, Internet traffic description, Industrial supervision of gear-wheel, Speech recognition, Computer graphics and multifractal analysis. Literature survey reveals the use of wavelet based denoising in a number of fields including biomedical, seismic, signal processing and so on[3-10].

10. CONCLUSION

The main objective of the work described in this paper is to describe a robust denoising technique of the signal. The introduced wavelet based signal analysis and coefficient thresholding methods produce a denoised signal which is more suitable for further analysis.

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