Empirical Correlation for Flow Conductance in Conductance based Modeling of Fluid Flow in Microchannels

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ABSTRACT

The electronic industry is looking at microchannel based MEMS technology to resolve the issue of increasing heat dissipation from electronic circuits. The internal flow configuration represents a convenient geometry for heat transfer between fluids used in chemical processing, environmental control, and energy conversion technologies. The rapid development of MEMS and micro-fluidic devices has generated an increasing demand for under- standing of fluid flows in micro- channel structures. In order to fabricate such micro devices effectively, it is extremely important to understand the fundamental mechanisms involved in fluid flow and heat transfer characteristics in microchannels. The fluid flow analysis in microchannels usually involves modeling the flow using continuity, momentum and energy equations and then solving them by analytical or numerical methods depending on the complexity of the problem. In this work, we use an alternate approach to model the flow through a microchannel, relating the quantities of interest i.e. volume flow rate, pressure drop across the channel and hydraulic diameter. We use a general electrical analogy of the fluid flow to model the flow through the microchannel. Based on this model, we derive an empirical correlation for parameter, termed as, Flow Conductance which represents the ease with which fluid can flow through a microchannel. It depends on channel geometry and fluid property. Experimental results of Mala et al. have been used to derive the correlation. The correlation is derived for the trapezoidal microchannel. Using the empirical correlation, flow parameters like pressure drop, Reynolds Number have been calculated. The results for flow parameters obtained by using the flow conductance model show good agreement with the experimental results. This suggests, the model could be used for analyzing the flow through the microchannels.

Keywords: Fluid Flow, Trapezoidal Microchannels

1. INTRODUCTION

Flow conductance, or conductance, of a microchannel can be interpreted as a property which will govern how easily the fluid can flow across the microchannel. More the conductance, lesser will be the driving pressure drop required to make the fluid flow through the microchannel. Moreover, it varies according to the geometrical parameters. Thus it is necessary to study dependence of conductance on the geometry of the microchannel. Conductance for simple geometries like circle or rectangle can be easily determined using well described analytical methods in fluid flow literature[1,2]. However as the geometry gets complicated, analytical methods are not suitable to obtain exact solutions. A good approximation can be obtained by numerical methods available, which again require tedious mathematical manipulations. Here we have used the experimental results to derive an empirical correlation for conductance based on hydraulic diameter for trapezoidal microchannel using water as working fluid.

2. THEORETICAL BACKGROUND

Descriptive details about fluid flow in channels and pipes can be found in various texts [1,2,3,4].Steady state fully developed flow in rectangular channel is given by solving the reduced Navier-Stokes equation as under:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z} = -\left(\frac{1}{\mu}\right)\left(\frac{\partial p}{\partial x}\right). \qquad (1)$$

Where

$$\frac{\partial p}{\partial x} = 0. \qquad (2)$$

Where u is the velocity along the channel, y and z are along the width and depth of the microchannel and x is along the length. Solving this equation for u and determining the volume flow rate Q, gives the relation between volume flow rate and pressure gradient as follows:

$$Q=\left(\frac{4ba^3}{3\mu}\right)\left(-\frac{\partial p}{\partial x}\right)\left[1-\left(\frac{192a}{\pi^5 b}\right)\sum_{1=1,3,5..}\left(\frac{\tanh\left(\frac{i\pi b}{2a}\right)}{i^5}\right)\right]. \qquad (3)$$

Where 2a and 2b are width and depth of the channel respectively. Equation 3 can also be written as:

$$Q = C_1\left(-\frac{dp}{dx}\right). \qquad (4)$$

Where quantity C_1 is a function of channel geometry and fluid viscosity only. Its constant for a given channel and fluid. This constant is termed as flow conductance or conductance of the channel. Equation 4 is analogous to the electrical relation between current I, voltage V and resistance R. Here Q is analogous to I, (dp/dx) to V and C_1 is analogous to reciprocal of R. The equation can be used in general to represent the fluid flow in a channel of any arbitrary cross-section. However, it is not easy to determine C_1 analytically as the cross-section shape gets complicated. Numerical methods are available but require tedious mathematical derivations.

Nevertheless, knowing the fact that C_1 is a function of channel geometry, an expression for C_1 can be derived using experimental results. Here we provide an experimentally determined formula for conductance C_1.

In this study, hydraulic diameter D_h of a channel is chosen as the critical geometric dimension which can be used as a basis for comparison between channels of different shapes(circular, rectangular, trapezoidal etc.), and also different sizes within the same shape. Further, D_h is the characteristic length used to determine Re in most literature[5,6,7,8,9]. Hence D_h is used as the parameter to describe geometric dependence of C_1.

3. FLOW CONDUCTANCE CORRELATION

The correlation between conductance C_1 and hydraulic Diameter D_h has been obtained using experimental results of Mala, Li et al[10]. In their experiment, fluid used was water and channel cross-section shape was trapezoidal. Following is the geometrical specifications of the channel used in their study:

Table 1: Microchannel specifications of Mala et al.[10]

Channel no.	Top, a	Bottom, b	Depth, h	D_h
	[micron]	[micron]	[micron]	[micron]
1	405.27	360	28.06	51.3
2	148.33	94.8	44.44	62.3
3	162.11	105	45.47	64.9
4	237.01	66.1	109.77	114.5
5	318.81	150	113.84	142
6	523.2	356	111.14	168.9

Fig. 1 gives the experimental data from Mala et al.[10] as a plot between pressure gradient vs. Re.

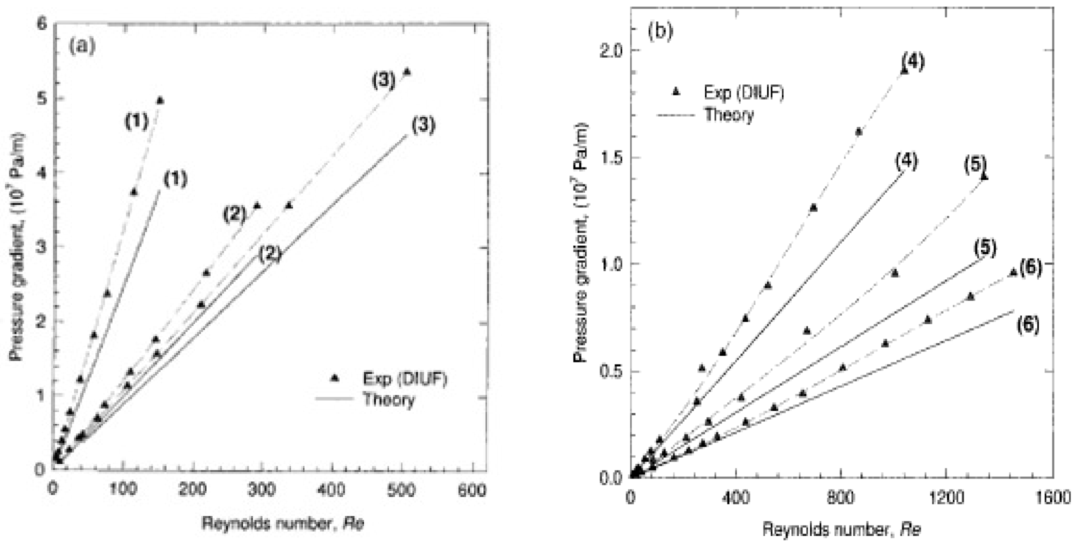

Fig 1. Pressure gradient vs. Re from Mala et al.[10] for different D_h

Using the experimental results of Mala et al.[10], Conductance C_1 was calculated at each different D_h. The results are given as below:

Table 2: Conductance C_1 for different microchannels of Mala et al[10]

D_h	C_1
[Micron]	[10^{-15} m^4/Pa-s]
51.3	0.656526267
62.3	0.69845335
64.9	0.866641277
114.5	8.738603168
142	20.41070045
168.9	48.06243117

The graph between C_1 and D_h is plotted as following:

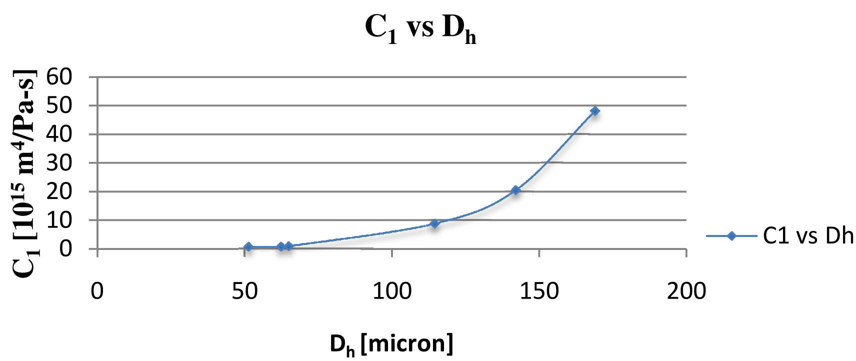

Fig. 2: Graph C_1 vs. D_h

As can be seen, C_1 increases as D_h increases. From the above graph, following emperical relation was determined between C_1 and D_h for a Trapezoidal microchannel:

$C_1 = A0 + A1*D_h^1 + A2*D_h^2 + A3*D_h^3 + A4*D_h^4$. (5)

Where coefficients have values:

A0=30.57196, A1= -1.45437, A2=0.02484, A3= -1.79737E-4, A4=5.16643E-7

D_h to be used in the formula should be in microns. Flow parameters calculated using value of conductance obtained from this relation yield a good prediction of experimental results.

4. RESULTS AND DISCUSSION

Results are given below for comparison of three flow parameters pressure gradient, Re and f.Re.

Table 3: Comparison of Pressure Gradient, [Pa/m]

D_h [micron]	Pressure Gradient Expt Mala et al.	Pressure Gradient New Model	Percent difference
51.3	49580200	48979690.47	-1.21
62.3	35679300	38404169.06	7.64
64.9	53469900	59082345.09	10.50
114.5	3536700	3631550.769	2.68
142	2538200	2528515.434	-0.38
168.9	1864500	1867828.397	0.18

Table 4: Comparison of Re

D_h [micron]	Re Expt Mala et al.	Re New Model	Percent difference
51.3	149	148	-0.40
62.3	279	282	0.89
64.9	486	490	0.77
114.5	251	248	-1.15
142	290	288	-0.67
168.9	323	321	-0.58

Table 5: Comparison of f.Re

D_h [micron]	f.Re Expt Mala et al.	f.Re New Model	Percent difference
51.3	84.88	90.278	6.36
62.3	60.696	60.822	0.21
64.9	61.624	62.296	1.09
114.5	52.084	49.57	-4.83
142	55.412	52.195	-5.81
168.9	65.196	57.303	-12.11

The model has also been validated with data of Han et al[11]. The results are given below.

Table 6: Comparison of Pressure drop, Model vs. Han et al [11]

D_h	Pressure Drop Model	Pressure Drop Experiment	Percent difference
[Micron]	[Pa/m]	[Pa/m]	
60	357381.6666	500000	-28.5237
70	444671.7463	500000	-11.0657
80	406902.2128	500000	-18.6196

The difference in values could be attributed to the uncertainty of measurement for various parameters and also the accuracy of data points obtained from plots. The larger deviation from results of Han et al.[11] could be due to difference in aspect ratios for the same D_h but needs to be investigated. Overall, the correlation obtained can be used to determine the value of conductance based on hydraulic diameter of a trapezoidal microchannel using water as working fluid.

5. CONCLUSION AND FUTURE SCOPE

The present work shows good approximation of the fluid flow parameters calculated using the flow conductance model. The results have been compared with the experimental results of Mala et al [10] and show good agreement, with average difference well below 10% from experimental values of all parameters used for comparison. Thus this model can be used in fluid flow analysis in microchannels and pipes in general.

In the current work, the study has been limited to trapezoidal channel with water as working fluid. However, the scope can be extended to include other different cross-section shapes of the microchannel as mentioned in various texts[3]. Empirical correlations for different geometries of the microchannel cross section could be derived in a similar way from the experimental results available in literature.

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