Cost-Benefit Analysis of a Cold Standby System with Two-Stage Repair and Waiting Time Subject to Arrival Time of the Server

Ashish Kumar[1], Monika Saini [2]

[1,2]*Department of Mathematics, School of Basic Sciences and Humanities,*
Manipal University Jaipur, Jaipur-303007, Rajasthan, India

Abstract: **The main objective of this investigation is to carry out the cost-benefit analysis of a two-unit cold standby system using the concepts of arrival time of the server, normal failure and two-stage repair of a failed unit. Here the concept of waiting time with two-stage repair for a two-unit cold standby system is discussed. There is a single server who takes some time to visit the system to do repair of the unit. The repair process is divided into two stages. In the first stage, the repairing process of the unit is started but it does not get completed; instead the process is completed in the second stage. The elapsed time between two stages is called the waiting time. The failure time of the unit follows negative exponential distribution while the distribution of repair times and waiting time of the unit are taken as arbitrary with different probability density functions. The unit works as new after repair and preventive maintenance. All random variables are statistically independent. Switch devices are perfect. The system has been analyzed stochastically in detail using semi-Markov process and regenerative point technique. The behavior of some important measures of system effectiveness has been observed numerically with respect to failure rate by giving particular values to other parameters.**

Keywords: **Cold Standby System, Arrival Time of the Server, Waiting Time, Two-Stage repair and Profit Analysis.**

1. INTRODUCTION

Two-unit cold standby redundant systems have been extensively studied by several authors such as Kumar and Malik (2012), Sridharan and Mohanavadivu (1998) and Goel and Sharma (1989) in the past by a common assumption of single server. Singh and Singh (1989) carried out the profit of a cold stand by system with provision of rest. A two-unit cold standby system is considered. The failure rate of a unit is a constant and the repair time distribution is a two-stage Erlang distribution is presented by Shandrasekhar et al. (2004). Mokaddis and El-Said (1990) developed a stochastic model for availability function and the mean time to the first failure for two models of a cold standby redundant system with two different types of repair. El-Said and El-Sherbeny (2005) considered the reliability of two units cold standby system with single repair and additional preventive maintenance of operative and the standby units. El-Said and El-Sherbeny (2010) developed a stochastic model for a cold standby system with two stage repair and waiting time.

In the present paper the idea of waiting time with two- stage repair for a two-unit cold stand by system is discuss subject to arrival time of the server. There is a single server who takes some time to visit the system to do repair of the unit. The repair process is divided into two stages. In the first stage, the repairing process of the unit is started but it does not get completed; instead the process is completed in the second stage. The elapsed time between two stages is called the waiting time. The failure time of the unit follows negative exponential distribution while the distribution of repair times and waiting time of the unit are taken as arbitrary with different probability density functions. The unit works as new after repair and preventive maintenance. All random variables are statistically independent. Switch devices are perfect. The system has been analyzed stochastically in detail using semi-Markov process and regenerative point technique. The behavior of some important measures of system effectiveness has been observed numerically with respect to failure rate by giving particular values to other parameters.

2. NOTATIONS

λ	Constant failure rate of the unit
$h(t)$ / $H(t)$	pdf / cdf of the waiting time
$g(t)$ / $G(t)$	pdf / cdf of repair time of the server
$w(t)$ / $W(t)$	pdf / cdf of arrival time of the server
$q_{ij}(t)$/ $Q_{ij}(t)$:	pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]
pdf / cdf	Probability density function/ Cumulative density function
$q_{ij.kr}(t)$/$Q_{ij.kr}(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to

	a failed state j visiting state k, r once in (0, t]
$\mu_i(t)$	Probability that the system up initially in state S_i ε E is up at time t without visiting to any regenerative state
$W_i(t)$:	Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
m_{ij}	Contribution to mean sojourn time $\mu_i(t)$ in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and m_{ij} = $\int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
® /©	Symbol for Laplace-Stieltjes convolution/Laplace convolution
~ / *	Symbol for Laplace –Steiltjes Transform (LST) / Laplace Transform (LT)

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \text{ as} \tag{1}$$

$p_{01}=1= p_{86} = p_{62}= p_{72} = p_{45}= p_{56}$, $p_{14}= \dfrac{\lambda}{\gamma+\lambda}$,

$p_{12} = \dfrac{\gamma}{\gamma+\lambda} = p_{12.456}$, $p_{28} = \dfrac{\lambda}{\beta+\lambda} = p_{22.8,6}$,

$$p_{23} = \frac{\beta}{\beta+\lambda}, \ p_{32.7} = p_{37} = \frac{\lambda}{\theta+\lambda}, p_{30} = \frac{\theta}{\theta+\lambda} \tag{2}$$

It can be easily verified that $p_{01}= p_{14}+p_{12} = p_{23}+ p_{28} = p_{30}+p_{37}= p_{45}= p_{56}= p_{62}= p_{72} = p_{86} = 1$

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{\lambda},\ \mu_1 = \frac{1}{\gamma+\lambda},\ \mu_2 = \frac{1}{\beta+\lambda},\ \mu_3 = \frac{1}{\theta+\lambda},\ \mu_1' = \frac{1}{\gamma},\ \mu_2' = \frac{1}{\beta},\ \mu_3' = \frac{1}{\theta}$$

4. SYSTEM MODEL DESCRIPTION

In this section, the two-unit cold standby redundant system is described. Through semi-Markov assumption, the recurrence equations are obtained for the analysis of state probabilities. The states of the system according semi-Markov process and regenerative point technique are as follows:

State 0: initial state, one unit works, one unit in standby, and the system is working

State 1: operative unit fails and waiting for repair, cold standby unit becomes operative; and the system is working

State 2: failed unit is under first stage repair, other unit is operative and the system is working

State 3: failed unit is under second stage repair, other unit is operative and the system is working

State 4: First failed unit continuously waiting for repair, second failed unit is also waiting for repair, and the system failed

State 5: First failed unit is under first stage repair, second failed unit is continuously waiting for repair, and the system failed

State 6: First failed unit is under second stage repair, second failed unit is continuously waiting for repair, and the system failed

State 7: First failed unit is continuously under second stage repair, second failed unit is waiting for repair, and the system failed

State 8: First failed unit is continuously under first stage repair, second failed unit is waiting for first stage repair, and the system failed

The states S0, S1, S2 and S3 are regenerative states while S4, S5, S6, S7 and S8 are non-regenerative states. Thus E = {S0, S1, S2, S3}.

5. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)} © A_j(t) \tag{3}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state, we have is

$$M_0(t) = e^{-\lambda t}, \; M_1(t) = e^{-\lambda t}\overline{W(t)}, \; M_2(t) = e^{-\lambda t}\overline{H(t)},$$
$$M_3(t) = e^{-\lambda t}\overline{G(t)} \tag{4}$$

Taking LT of above relations (3) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} s\,A_0^*(s) = \frac{N_1}{D_1}, \text{where} \tag{5}$$

N_1 = [(μ_0+μ_1 p_{01}) (1-$p_{22.8,6}$- p_{23} $p_{32.7}$)+ (p_{12} + $p_{12.4,5,6}$)p_{01}(μ_2+μ_3 p_{23})]

and

D_1 = [(μ_0+ μ_1' p_{01}) (1-$p_{22.8,6}$- p_{23} $p_{32.7}$)+ (p_{12} + $p_{12.4,5,6}$)p_{01}(μ_2' + $\mu_3'$$p_{23}$)]

6. BUSY PERIOD ANALYSIS FOR SERVER

Let $B_i(t)$ be the probability that the server is busy in repairing of the system (unit) at an instant 't' given that the system entered state i at t = 0. The recursive relations for $B_i(t)$ are as follows:

$$B_i(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j(t) \tag{6}$$

Where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to repairing of the unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2 = e^{-\lambda t}\overline{H}(t), \; W_3 = e^{-\lambda t}\overline{G}(t)$$

Taking LT of above relations (6) and solving for B_0^* (s). The time for which server is busy due to repair is given by

$$B_0(\infty) = \lim_{s\to 0} sB_0^*(s) = \frac{N_2}{D_1}, \text{ where}$$

$N_2 = [W_2^*(0) + P_{23}W_3^*(0)]$ [2(p_{12} + $p_{12.4,5,6}$)p_{01}] and D_1 is already defined.

7. EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_j(t)\right] \tag{7}$$

Where j is any regenerative state to which the given regenerative state i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relation (7) and solving for $\tilde{N}_0(s)$. The expected number of visit per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\,\tilde{N}_0(s) = \frac{N_3}{D_1}, \text{ where} \tag{8}$$

N_3 = [p_{12} p_{01} (1-$p_{22.8,6}$- p_{23} $p_{32.7}$)+ (p_{12} + $p_{12.4,5,6}$)$p_{01}p_{23}$ $p_{12.4,5,6}$] and D_1 is already defined.

8. COST-BENEFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0 - K_2N \tag{9}$$

K_0 = Revenue per unit up-time of the system
K_1 = Cost per unit time for which server is busy due repair
K_2 = Cost per unit time visit by the server

9. CONCLUSION

By considering a particular case, $g(t) = \theta e^{-\theta t}$, $h(t) = \beta e^{-\beta t}$ and $w(t) = \gamma e^{-\gamma t}$, the numerical results for some system effectiveness measures are obtained for the system under study. The numerical results for availability and profit are obtained with respect to failure rate (λ) rate for fixed values of parameters as shown respectively in table 1 and 2. It is revealed that Availability and profit increase with the increase of repair rate (θ) and arrival rate of the server (γ). But the value of these measures decrease with the increase of failure rate (λ) and waiting time (β). Thus finally it is concluded that in a cold standby system with arrival time of the server can be made reliable and economical to use

i.) By reducing the waiting time between both stages of repair.
ii. By increasing the repair rate.

REFERENCES

[1] Goel, L.R. and Sharma, S.C., Stochastic analysis of a two-unit standby system with two failure modes and slow switch. Microelectronics and Reliability, 29(4),1989, pp. 493-498.

[2] Khaled, M. El-said and Mohamed, S. El-Sherbeny, Profit analysis of a two unit cold standby system with preventive maintenance and random change in units. *J. Mathematics and Statistics*, 1,2005, 71-77.

[3] Khaled, M. El-Said and Mohamed, S. El-Sherbeny, Stochastic analysis of a two-unit cold standby system with two stage repair and waiting time. Sankhya, 72-B,2010,pp.1-10.

[4] Malik, S.C. and Kumar, A., Reliability modeling of a computer system with priority to s/w replacement over h/w replacement subject to MOT and MRT. *International Journal of Pure and Applied Mathematics*, 80(5),2012,pp. 693-709.

[5] Mokaddis, G., S. and Khaled, M. E., Two models for two-dissimilar unit cold standby redundant system with partial failure and two types of repairs. *Microelectronic Reliability*, 30,1990,pp. 431-451.

[6] Shandrasekhar, P., Natarajan, R. and Yadavalli, S.S.V., A study on a two-unit standby system with Erlangian repair time. *Asia Pacific Journal of Operational Research*, 21,2004, pp. 271-277.

[7] Singh, S.K. and Singh, R.P., Profit evaluation of a two-unit cold standby system with a provision of rest. *Microelectronics and Reliability*, 29(5),1989,pp. 705-709.

[8] Sridharan, V. and Mohanavadivu, P., Stochastic behaviour of a two-unit standby system with two types of repairmen and patience time. *Math. Comput. Modelling*, 28,1998, pp. 6371.

Table: 1. Effect of Arrival Rate of the Server and Various Repair Policies on Availability with respect to Failure Rate (λ)

Λ	γ=.07,θ=.5,β=.8	γ=.07,θ=.5,β=1.0	γ=.07,θ=.7,β=.8	γ=.04,θ=.5,β=.8
0.01	0.998368	0.995592	0.999834	0.968473
0.02	0.964962	0.962194	0.969458	0.891677
0.03	0.91888	0.917306	0.926686	0.811333
0.04	0.869181	0.869196	0.880019	0.73819
0.05	0.820091	0.821728	0.833502	0.674214
0.06	0.773494	0.776633	0.788999	0.61886
0.07	0.73012	0.734587	0.747285	0.570972
0.08	0.690138	0.695753	0.708596	0.52937
0.09	0.653449	0.660043	0.672896	0.49302
0.1	0.619834	0.627259	0.640026	0.461059

Table: 2. Effect of Arrival Rate of the Server and Various Repair Policies on Profit Function with respect to Failure Rate (λ)

Λ	γ=.07,θ=.5,β=.8	γ=.07,θ=.5,β=1.0	γ=.07,θ=.7,β=.8	γ=.04,θ=.5,β=.8
0.01	4983.855	4971.489	4991.219	4835.746
0.02	4811.921	4800.51	4834.476	4449.015
0.03	4578.45	4573.558	4617.588	4046.152
0.04	4328.02	4331.401	4382.349	3680.012
0.05	4081.352	4093.033	4148.566	3360.051
0.06	3847.6	3866.895	3925.301	3083.367
0.07	3630.26	3656.25	3716.285	2844.089
0.08	3430.076	3461.824	3522.585	2636.279
0.09	3246.486	3283.128	3343.967	2454.738
0.1	3078.357	3119.132	3179.583	2295.138