Upshot of Magnetic Field on Free and Force Convection FLOW in a Vertical Channel with Heat Transfer

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Abstract: **The problem of applied magnetic field on free convective flow in a vertical channel with heat transfer is taken for discussion to understand the magnetic upshots. A uniform magnetic field is applied normal to the plate. The equation governing the fluid flow and heat transfer have been solved subject to the relevant basic equations and boundary conditions. It is noticed that the magnetic field M has considerable effect on u, θ, τ_W, τ_1, Nu_1 . All the profiles increases with M. The behaviour of the fluid velocity U for α > 0 is reversed to the normal velocity V for α = 0, however both are diminishing sharply.**

1. INTRODUCTION

In recent years the problem of the free convective flow of a viscous fluid over a uniformly moving long vertical wavy wall and a parallel flat wall has got a threadbare response from research scholars with a view to understanding its application in transportation cooling of re-entry vehicles and rocket boosters, cross- hatching on ablative surfaces and film vaporization in combustion chambers. Lekoudias, Nayfeh and Saric are pioneering contributor in this field because they have discussed linear analysis of compressible boundary layer flows over a wavy wall. The Rayleigh problem for a wavy wall and amplitude wall waviness upon the stability of the laminar boundary layer have been discussed by lessen and Gangwani (1976). The free convection heat transfer in viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall was presented by Vajravelu and Sastri (1978). Free convection heat transfer in a viscous incompressible fluid between a uniformly moving long vertical wavy wall and a parallel flat wall have been discussed by S. Ahmed et al. (2002). The aim of the present exploration is to find out the effects on the uniform magnetic field applied normal to the direction of main flow with a view to make an extension of the study done so far by S. Ahmed et al. (2002) in this field.

2. BASIC EQUATIONS

We consider the two dimensional steady laminar free convective MHD flow along the vertical channel as shown in Fig. 1.

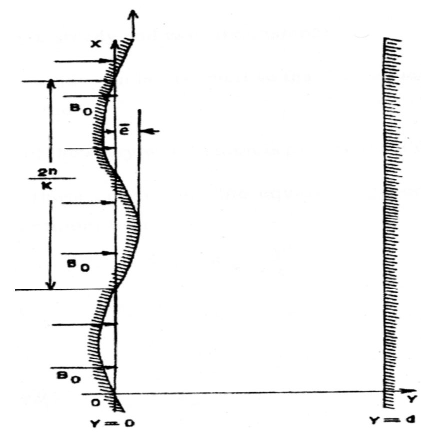


Fig. 1. Flow Configuration

In the channel, $\bar{X}$-axis is taken along the flat wall and the $\bar{Y}$-axis is taken perpendicular to it. Moreover, $\bar{T}_W$ and $\bar{T}_1$ are the constant temperatures at the respective wavy wall $\bar{Y}=\bar{\varepsilon}\cos\bar{k}\bar{x}$ and the flat wall $\bar{Y}$= d.

In view of this, we consider that:

1) Except density in the resilience force, fluid properties are constant.
2) In energy equation, the viscous and magnetic dissipative are meant to be ignored.
3) In context volumetric heat either source or sink does not change its energy equation.
4) The flow is laminar, balanced and two dimensional.
5) The magnetic Reynolds number is small so that the stimulated magnetic field can be abandoned.
6) The wave length of the wavy wall, which is proportional to K^{-1}, is large.

Under these conditions, the equations governing the fluid motion in non-dimensional form are:

$uu_x + vu_y = -p_x + u_{xx} + u_{yy} - 1\frac{gd3}{v2} - Mu$ (4.2.1)

$uv_x + vv_y = -P_y + V_{xx} + V_{yy},$ (4.2.2)

$u_x + v_y = 0,$ (4.2.3)

$p(u\theta_x + v\theta_y) = \theta_{xx} + \theta_{yy} + \alpha$ (4.2.4)

Subject to the boundary conditions:

$$\left.\begin{array}{l} u = A,\ v = 0,\ \theta = 1,\ \text{at } y = \varepsilon\cos\lambda x \\ u = v = 0,\ \theta = m \quad \text{at } y = 1 \end{array}\right\} \quad (4.2.5)$$

The non-dimensional quantities introduced in the above equations are:

$x = \bar{x}/d, \quad y = \bar{y}/d, \ u = d\,\bar{u}/\upsilon, \ v = d\,\bar{v}/\upsilon,$

$p = \bar{p}/\rho(\upsilon / d)^2, \quad \theta = (\bar{T} - \bar{T}_s) / (\bar{T}_w - \bar{T}_s)$

$m = (\bar{T}_1 - \bar{T}_s) / (\bar{T}_w - \bar{T}_s)$, the wall temperature ratio,

$\alpha = Qd^2 / k\,(\bar{T}_w - \bar{T}_s)$, the heat source / sink parameter,

$P = \mu Cp / k$, the Prandtl number,

$\varepsilon = \bar{\varepsilon}$ /d, the amplitude parameter

$\gamma = \bar{k}$ /d, the frequency parameter,

$G = d^3 g\beta\,(\bar{T}_w - \bar{T}_s) / \upsilon^2$, the grashof number or free convection parameter,

$M = \sigma B^2_0 d^2/p\upsilon$, the Hartmann number

Where $\bar{u}$, $\bar{v}$ are velocity components, $\bar{p}$ fluid pressure, ρg the buoyancy force, Q the constant

Heat addition or absorption, B_0 the magnetic induction and the other symbols have their usual meanings.

Under the perturbations technique, let us assume that the flow field and temperature field are to be:

$$\left.\begin{array}{l} u(x, y) = u_0(y) + u_1(x, y), \\ v(x, y) = v_1(x, y) \\ p(x, y) = p_0(x) + p_1(x, y), \\ \theta(x, y) = \theta_0(y) + \theta_1(x, y), \end{array}\right\} \quad (4,2,6)$$

Where the perturbations u_1, v_1, p_1 and θ_1 are small compared with the mean or the zeroth-order quantities.

On using (4.2.5), the equations (4.2.1) to (4.2.4), transformed to the non-dimensional form:

For zeroth order :

$u_0^{//} - Mu_0 + G\,\theta_0 = C, \ = -\alpha\theta_0''$ (4.2.7)

And for the first order:

$u_o u_{1,x} + v_1 u'_0 = -P_{1,x} + u_{1,xx} + u_{1,yy} + G\theta_1 - Mu_1$ (4.2.8)

$u_o v_{1,x} = -P_{1,y} + v_{1,xx} + v_{1,yy}$ (4.2.9)

$u_{1,x} + v_{1,y} = 0$ (4.2.10)

$P(u_o\theta_{1,x} + v_1\theta'_0) = \theta_{1,xx} + \theta_{1,yy}$ (4.2.11)

Where $c = (\frac{\partial p_0}{\partial x} - p_s)$ is the constant pressure gradient term and is taken equal to zero following Ostrach (1952) and subscript 's' stands for static fluid condition.

With the help of (4.2.6), the boundary conditions (4.2.5) can be split up into the following two parts:

$y = 0 : u_o = A,\ \theta_0 = 1$

$y = 1 : u_o = 0\ \theta_0 = m$ (4.2.12)

and

$u_1 = -\text{Re}\,\{\varepsilon\, u'_0 e^{i\lambda x}\},\ v_1 = 0,\ \theta_1 = -\text{Re}\,\{\varepsilon\theta'_0 e^{i\lambda x}\}$ at $y = 0$

$u_1 = 0,\ v_1 = 0,\ \theta_1 = 0$ at $y = 1$ (4.2.13)

Where the prime denotes differentiation w.r.t y.

3. METHOD OF SOLUTION FOR BOTH THE ORDERS

With the help of (4.2.12), the zeroth order solutions from the equations (4.2.7) have been obtained but are not presented here due to sake of brevity.

To find the first order solutions from the equations (4.2.8) to (4.2.11), let us introduce the stream function Ψ_1, defined by

$u_1 = -\bar{\Psi}_{1,y}\ v_1 = \bar{\Psi}_{1,x}$ (4.3.1)

Where, $\bar{\Psi}_1(x,y) = \varepsilon e^{i\lambda x}\Psi(y),\ \theta_1(x,y) = \varepsilon e^{i\lambda x}t(y)$

For small value of 1 (or K << 1), we consider:

$$\Psi(\lambda,y) = \sum_{i=0}^{2} \Psi_i,\ t(\lambda,y) = \lambda^i t_i \sum_{i=0}^{2} \quad (4.3.2)$$

On using (3.1) and (3.2) into (2.8) – (2.11), to the order λ^2, the following sets of ordinary differential equations and corresponding boundary conditions :

$$\Psi^{IV}_0 - M\,\Psi_0'' = Gt_0',\ t_0'' = 0 \quad (4.3.3)$$

$$\left.\begin{aligned} &\Psi^{IV}_1 - M\,\Psi_1'' = i(u_0\Psi_0'' - \Psi_0 u_0'') + Gt_1 \\ &t_1'' = iP\,(u_0 t_0 + \Psi_0\,\theta_0') \end{aligned}\right\} \quad 4.3.4)$$

$$\left.\begin{aligned} &\Psi^{IV}_2 - M\,\Psi_2'' = 2\Psi_0'' + i(u_0\Psi_1'' - \Psi_1 u_0'') + Gt_2'' \\ &t_2'' = iP\,(u_0 t_1 + \Psi_1\,\theta_0') + t_0 \end{aligned}\right\} \quad (4.3.5)$$

$$\left.\begin{aligned} &\text{and } \Psi_0' = u_0',\ \Psi = 0,\ t_o = -\theta'_0 \text{ at } y = 0 \\ &\Psi_{0'} = 0,\ \Psi_o = 0,\quad t_o = 0 \text{ at } y = 1 \end{aligned}\right\} \quad (4.3.6)$$

$$\left.\begin{aligned} &\Psi_i' = \Psi_i = t_i = 0 \quad \text{at } y = 0 \\ &\Psi_i' = \Psi_i = t_i = 0 \text{ at } y = 1 \end{aligned}\right\} \text{for } i \geq 1 \quad (4.3.7)$$

With the help of (4.3.6) to (4.3.7), the solutions of the equations (4.3.3) to (4.3.4) have been obtained but not presented here due to sake of brevity. From these solutions, due to waviness of the wall, the expressions for u_1, v_1 and θ_1 are the first order solutions or the disturbed parts and they can be put into the form :

$$\left.\begin{aligned} &u_1 = \varepsilon[\Psi_i' \sin\lambda x - \Psi_r'\cos\lambda x], \\ &v_1 = -\varepsilon\lambda[\Psi_r \sin\lambda x - \Psi_i\cos\lambda x], \\ &\theta_1 = \varepsilon[t_r\cos\lambda x - t_i \sin\lambda x], \\ &\text{where}\quad \Psi = \Psi_1 + i\Psi_i\quad t = t_r + it_i \end{aligned}\right\} \quad (4.3.8)$$

4. SKIN FRICTION AND HEAT TRANSFER COEFFICIENT AT THE WALLS

The non-dimensional skin friction at the wavy wall y = $\varepsilon\cos\lambda x$ and at the flat wall y = 1 are respectively:

$$\tau_u = \tau^0_0 + \varepsilon\cos\lambda x\,[u_0''(0) + \bar{u}'(0)], \quad (4.4.1)$$

$$\tau_1 = \tau^0_1 + \varepsilon\,\bar{u}_1(1)\cos\lambda x, \quad (4.4.2)$$

Where $\tau^0_0 = u_0'(0)$ and $\tau^0_1 = u_0'(1)$ are the zeroth order skin frictions at the walls and

$u_1(x, y) = \varepsilon e^{i\lambda x}\bar{u}_1(y)$ and $v_1(x,y) = \varepsilon e^{i\lambda x}\bar{v}_1(y)$

The non-dimensional Nusselt Nu at the wavy wall y= $\varepsilon\cos\lambda x$ and at the flat wall y = 1 respectively:

$$Nu_w = Nu^0_0 + \varepsilon\cos\lambda x\,[\theta_0''(0) + t'(0)], \quad (4.4.3)$$

$$Nu_1 = Nu^0_1 + \varepsilon\cos\lambda x\,[t'(1)], \quad (4.4.4)$$

where $Nu^0_0 = \theta_0'(0)$ and $Nu^0_1 = \theta_0'(1)$ are the zeroth order skin frictions at the walls.

5. OUTCOME ANDDISCUSSION

All Mathematical computation are done in order to $\varepsilon = 0.05$, $\lambda = 0.5$. The action of the velocity profiles u, v and the temperature profile θ are illustrated in figs. 2 – 4 and they have been schemed against y as abscissa.

When $\alpha = 5$, the fluid velocity u increase as M (I, II, III) and A (I, IV, V), while it drop off with the augment of P (I, VI, VII). When $\alpha = 0$, the normal velocity v reduces with the boost of M and A, whereas v raises as K. When $\alpha = -5$, the q increases as M and P, but θ decreases with the increase of A.

Figures 5 to 8 show the behaviour of the skin frictions and the Nusselt numbers at the channel walls and they have been designed against α as abscissa. When G=0, the skin friction τ_w raises as raises as M and A, whereas τ_1 reduces with the increase of P. When G=0, the heat transfer coefficient Nu_w decreases with the increase of M, while Nu_w increases as A and P. Atlength, when G = 10, the Nu_1 heaves as M and P, but Nu_1 drops with the enhancement of A.

6. INFERENCE

The above mentioned outlines have made the following inferences -

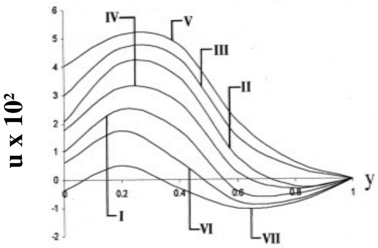
(i) The magnatic field M has considerably important effect on u, θ, τ_w, τ_1 and Nu_1. The outlines have augment inference.

(ii) The upshots of the parameters K, M, G on Nu_w for the air has been reversed to Nu_1 for the water.

(iii) The movement of the fluid velocity u for $\alpha>0$ is not being directed to the normal velocity v for $\alpha = 0$, however both are diminishing keenly.

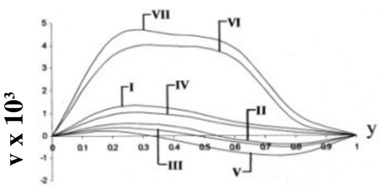
(iv) The result of v, θ, τ_w, Nu_w and Nu_1 is not being droped off with the P; but this result is reversed for u and τ_1.

(v) We see that u, τ_w, τ_1 and Nu_w raised with or without plate velocity A. While this result is reversed to v, θ and Nu_1.



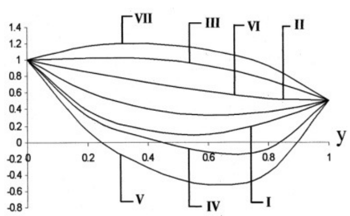
	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 2: Velocity profile u against y when m = 2, G =10, λx = 0



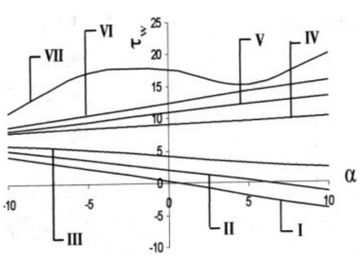
	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 3: Velocity profile V against y when m = 2, G =10, λx = $\frac{\pi}{2}$



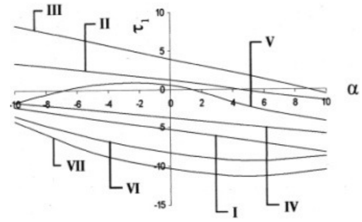
	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 4: Velocity profile V against y when m = 2, G =10, λx =$\frac{\pi}{2}$



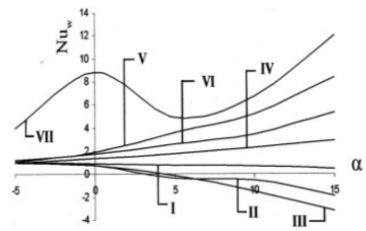
	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 5: Skin friction τ_w at y = 0 against α when m = 2, G = 0, λx = 0



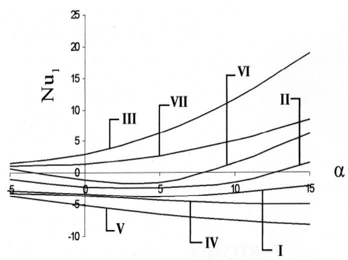
	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 6: Skin friction τ_1 at y = 1 against α when m = 2, G = 10, λx = 0



	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 7: Heat transfer coefficient Nu_1 at y = 0 against α when m = 2, G = 0, λx = 0



	M	A	P
I	1	0	0.71
II	7	0	0.71
III	15	0	0.71
IV	1	6	0.71
V	1	10	0.71
VI	1	0	7
VII	1	0	15

Fig. 8: Heat transfer coefficient Nu_1 at y = 1 against α when m = 2, G = 10, λx = 0

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