On m-Open Sets in Topology

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Abstract: **Minimal open sets or m-open sets for a topology are defined and investigated. They are found to form an Alexandroff space on X. Decompositions of open sets and continuity are provided using$m$-open sets.Also$m$-regularity and m-normalityare defined and studied. While Hausdorffness implies m-regularity, the product of normal spaces are found to be m-normal. 2010 Mathematics Subject Classification: Primary 54A05, 54A10, 54C08.**

Keywords and phrases: **Open set, Continuity, regularity.**
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1. INTRODUCTION

Now a days topological approaches are being investigated in a big way in various diverse field such as computer graphics, evolutionary theory, robotics etc.[6, 9, 16] to name a few. One such approach to computer graphics utilizes finite, connected order topological space[8]. In a finite topological space, the intersection of all open neighbourhoods of a point p is again an open neighbourhood of p, which is the smallest one. It is called the *minimal neighbourhood* of p. The topology of a finite space is completely determined by its minimal neighbourhoods. However, in a general framework of all topological spaces this is not true. Nevertheless, the sets which are realized as arbitrary intersection of open sets in topology are quite interesting. In this paper, we have made an investigation of all these type of sets. The m-open sets, as we call them, being a weaker form of open sets, are studied here in the light of other generalized form of open sets.

The m-open sets of a given topology on a set X again forms a topology on X. This is a departure from the fact that weaker forms of open sets usually form a "generalized topology"[4], not a topology. This development has led to few decompositions of open sets as well as decomposition of continuous mappings. A weaker form of regularity is defined using m-open sets. The product of m-regular spaces are found to be m-regular.

2. PRELIMINARIES

We recall some known definitions:

Definition 2.1 Let (X, τ) be a topological space. Then a subset A of (X, τ) is called,

i.)semi-open[10] if $A \subseteq cl\ int(A)$.
ii.)α-open [13] if $A \subseteq int\ cl\ int(A)$.
iii.)pre-open[11] if $A \subseteq int\ cl(A)$.
iv.)β-open[1] if $A \subseteq cl\ int\ cl(A)$.
v.)regular open (*regular closed* resp.,) [5] if $A = int\ cl(A)$ ($A = cl\ int(A)$ resp.,).

The complement of a semi-open (resp. α-open, pre-open, β-open) set is known as semi-closed (resp. α-closed, pre-closed, β-closed) set.

Definition 2.2 A subset$S$of a topological space$(X, \tau)$is said to be
*i.)*an A-set[14] if $S = U \cap C$, where U is open and C is regular closed.
*ii.)*a t-set[15] if $int(clS) = intS$.
iii.) a B-set[15] if there is an open set U and a t-set A in X such that $S = U \cap A$.

Let $f: X \to Y$ be a mapping, V be an arbitrary open set in Y. Then f is said to be*semi-continuous*[10] (resp. *pre-continuous*[11], α-*continuous*[12], β-*continuous*[14]) if $f^{-1}(V)$ is semi-open (resp. pre-open, α-open, β-open) in X. f is said to be A-continuous [14] (resp. B-continuous[15]) if $f^{-1}(V)$ is an A-set (resp. B-set) in X whenever V is open in Y. It is known that α-continuity implies pre-continuity and semi-continuity, A-continuity implies semi-continuity[14]. It can be shown that a subset S in X is open if and only if it is an A-set and an α-set[14] or equivalently, it is a pre-open set and B-set[15].

3. m-OPEN SETS

In this section, first we define m-open sets in a topology. It is shown that although, m-open sets are weaker form of open sets of the given topology, yet they also form a topology on their own.

Definition 3.1 Let(X, τ)be a topological space. A set$A \subseteq X$is called *minimal open or m-open* if Acan be expressed as intersection of a subfamily of open sets.

The collection of m-open sets of a topology (X, τ) is denoted by $\mathcal{M}$.

Clearly, every open set is m-open. In a finite space, open sets are the only m-open sets.

The following example gives an idea about the abundance of m-open sets.

Example 3.2 Let $X = \mathbb{N}$, the set of natural numbers, equipped with the cofinite topology. Then every subset of X is *m-open.*

Result 3.3 For a topological space (X, τ)
i.) $\emptyset, X \in \mathcal{M}$,
ii.) $\mathcal{M}$ is closed under arbitrary union,
iii.) $\mathcal{M}$ is closed under arbitrary intersection.

Proof. $i), iii)$ are obvious.

$ii)$ holds in view of the fact that $P(X)$, the power set of X forms a completely distributive lattice under union and intersection of sets.

Definition 3.4 Let (X, τ) be a topological space and $A \subseteq X$. Then minimal cover of A, denoted by $\mathcal{C}_m(A)$, is defined as $\mathcal{C}_m(A) = \cap\{U: U \in \tau, A \subseteq U\}$.

From the definition, it follows that $\mathcal{C}_m(A)$ is the smallest m-open set containing A.

Theorem 3.5 *Let (X, τ) be a topological space and A, B be subsets of X. Then the following hold:*
i.) $A \subseteq \mathcal{C}_m(A)$;
ii.) $\mathcal{C}_m(\mathcal{C}_m(A)) = \mathcal{C}_m(A)$;
iii.) If $A \subseteq B$ then $\mathcal{C}_m(A) \subseteq \mathcal{C}_m(B)$;
iv.) $\mathcal{C}_m(A \cup B) = \mathcal{C}_m(A) \cup \mathcal{C}_m(B)$;
v.) $\mathcal{C}_m(\emptyset) = \emptyset$.

Proof. Obvious from the definition of minimal cover of A.

This shows that $\mathcal{C}_m(A)$ is a closure operator. An operator similar to $\mathcal{C}_m$ was defined for generalized topological spaces in [3]. However the definition in [3] seems to erroneous or incomplete.

Theorem 3.6 *Let (X, τ) be a topological space. We define*

$$\mathfrak{T} = \{A \subseteq X: \mathcal{C}_m(A) = A\}.$$

Then $(X, \mathfrak{T})$ is a topological space and $\tau \subseteq \mathfrak{T}$.

Proof. Clearly $\emptyset, X \in \mathfrak{T}$. Let $A_i \in \mathfrak{T}$ where $i \in \wedge$, $A = \bigcup_{i\in\wedge} A_i$.

Since $A_i \subseteq \bigcup_{i\in\wedge} A_i$, thus $\mathcal{C}_m(A_i) \subseteq \mathcal{C}_m(\bigcup_{i\in\wedge} A_i)$ and hence $\bigcup_{i\in\wedge} \mathcal{C}_m(A_i) \subseteq \mathcal{C}_m(\bigcup_{i\in\wedge} A_i)$. Conversely, suppose that $x \notin \bigcup_{i\in\wedge} \mathcal{C}_m(A_i)$. Then $x \notin \mathcal{C}_m(A_i)$ for each $i \in\wedge$ and hence there exists an open set U_i containing A_i for each i, such that $x \notin U_i$.

Therefore $x \notin \bigcup_{i\in\wedge} U_i$, which contains $\bigcup_{i\in\wedge} A_i$. Hence $x \notin \mathcal{C}_m(\bigcup_{i\in\wedge} A_i)$. Therefore $\bigcup_{i\in\wedge} \mathcal{C}_m(A_i) = \mathcal{C}_m(\bigcup_{i\in\wedge} A_i)$. Hence $\bigcup_{i\in\wedge} A_i = \bigcup_{i\in\wedge} \mathcal{C}_m(A_i) = \mathcal{C}_m(\bigcup_{i\in\wedge} A_i) \supseteq \bigcup_{i\in\wedge} A_i$. Lastly we show that if $A, B \in \mathfrak{T}$ then $A \cap B \in \mathfrak{T}$. We know that $\mathcal{C}_m(A \cap B) \subseteq \mathcal{C}_m(A) \cap \mathcal{C}_m(B) = A \cap B$. But $A \cap B \subseteq \mathcal{C}_m(A \cap B)$ and hence $\mathcal{C}_m(A \cap B) = A \cap B$. Thus $\mathfrak{T}$ forms a topology.
Furthermore let $U \in \tau$, then $\mathcal{C}_m(U) = U$. Thus $U \in \mathfrak{T}$. Hence $\tau \subseteq \mathfrak{T}$.

Theorem 3.7 *Let (X, τ) be a topological space. Then $(X, \mathfrak{T})$ forms an Alexandroff space[2], that is, it is closed under arbitrary intersection also.*

Proof. Let $A_i \in \mathfrak{T}$ for each $i \in\wedge$ then $\mathcal{C}_m(\bigcap_{i\in\wedge} A_i) \subseteq \mathcal{C}_m(A_i) = A_i$ for each i. Thus $\mathcal{C}_m(\bigcap_{i\in\wedge} A_i) \subseteq \bigcap_{i\in\wedge} A_i$. Again $\bigcap_{i\in\wedge} A_i \subseteq \mathcal{C}_m(\bigcap_{i\in\wedge} A_i)$. Hence $\mathcal{C}_m(\bigcap_{i\in\wedge} A_i) = \bigcap_{i\in\wedge} A_i$. Therefore if $A_i \in \mathfrak{T}$ then $\bigcap_{i\in\wedge} A_i \in \mathfrak{T}$.

From **Theorem** 3.5 and **Theorem** 3.7 , one can observe that $\mathfrak{T}^c = \{A^c | A \subseteq X: \mathcal{C}_m(A) = A\}$ is also a topology.

The members of $\mathfrak{T}$ are called the minimal open or m-open sets of τ. If X is finite, then $\tau = \mathfrak{T}$.

In the following, we study the interrelationship of m-open sets with other existing notions and finally obtain a decomposition of open set.

4. A DECOMPOSITION OF OPEN SETS

The operator $\mathcal{C}_m$ defined in the previous section can be used to define new weaker forms of open sets. We show that these weaker forms provide decomposition of open sets as well as that of continuity.

Definition 4.1 Let (X, τ) be a topological space. A subset $S \subseteq X$ is said to be
i.) *$\mathcal{C}_m$-pre-open* if $S \subseteq int(\mathcal{C}_m(S))$,
ii.) *$\mathcal{C}_m$-t-set* if $int(S) = int(\mathcal{C}_m(S))$.
where int is the interior operator.

One can observe that every open set is $\mathcal{C}_m$-pre open set as well as $\mathcal{C}_m$-t-set. But converse is not true in general. In fact, we have the following example:

Example 4.2 Let $X = \mathbb{R}$, the set of real numbers with the usual topology. Take $A = (5,6) \cap \mathbb{Q}$. Then A is neither open nor semi-open and α-open set. But A is *$\mathcal{C}_m$-pre open-set as* $\mathcal{C}_m((5,6) \cap \mathbb{Q}) = (5,6)$.

Similarly if we take $B = (5,6]$, then B is $\mathcal{C}_m$-t-set but not open.

We can observe that if X is finite, then the class of $\mathcal{C}_m$-pre-open sets always forms a discrete topology. Because if X is finite then $\mathcal{C}_m(A)$ is an open set containing A.

Remark 4.3 A closed set need not be a $\mathcal{C}_m$-t-set.
We have the following example:

Example 4.4 Let$X = \{a, b, c, d\}$with topology$\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. If we take$A = \{d\}$. Then$A$is closed but not$\mathcal{C}_m$-*t-set.*

In **Example** 4.4, we see that $A = \{d\}$ is a t-set but not $\mathcal{C}_m$-t-set whereas $B = \{a, b, c\}$, being an open set is a $\mathcal{C}_m$-t-set, but not a t-set. Thus we can conclude that $\mathcal{C}_m$-t-set is independent of t-set.

Our next examples establish a fact that a $\mathcal{C}_m$-t-set is independent of open, semi-open, pre-open, α-open and β-open sets.

Example 4.5 Let$X = \mathbb{R}$, the set of real numbers with the usual topology. Take$A = (5,6) \cup \{\pi\}$. Then$A$is neither semi-open, pre-open, α-open and β-open set. But$A$ *is* $\mathcal{C}_m$-*t-set.*

Example 4.6 Let$X = \{a, b, c, d\}$with topology$\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. If we take$A = \{a, b, d\}$. Then $A$is α-open set and hence pre-open, semi-open and β-open set. But$A$is not$\mathcal{C}_m$-*t-set* because$\mathcal{C}_m(A) = X$.

Also a $\mathcal{C}_m$-t-set is independent from A-set and B-set. We have the following examples:

Example 4.7 Let$X = \{a, b, c, d\}$with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. If we take$A = \{b, c, d\}$, then $A$is aB-*set* because$A$is closed. But$A$is not a$\mathcal{C}_m$-*t-set.*

Example 4.8 Let$X = \{a, b, c, d\}$with topology $\tau = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}\}$. If we take$A = \{c, d\}$, then$A$is an$A$-*set.* But$A$is not a$\mathcal{C}_m$-*t-set.*

5. SIMILARLY, WE HAVE

Example 4.9 Let$X = \mathbb{R}$, the set of real numbers with the usual topology. Take $B = [0,1) \cup (1,2) \cup \{\pi\}$. Then$Bis\mathcal{C}_m$-*t-set.* But not a$B$-*set.* If we take$A = (5,6) \cup \{\pi\}isa\mathcal{C}_m$-$t$-set but not an$A$-*set* because$A$is not a semi-open set.

Hence a $\mathcal{C}_m$-t-set is independent from A-set and B-set.

Proposition 4.10 If $A, B$are two$\mathcal{C}_m$-t-set, then$A \cap B$is also a$\mathcal{C}_m$-t-set.

Proof. Let A, B be two $\mathcal{C}_m$-t-sets. Then $int(A \cap B) \subseteq int(\mathcal{C}_m(A \cap B)) \subseteq int(\mathcal{C}_m(A) \cap \mathcal{C}_m(B)) = int(\mathcal{C}_m(A)) \cap int(\mathcal{C}_m(B)) = int(A) \cap int(B) = int(A \cap B)$.

Therefore $int(A \cap B) = int(\mathcal{C}_m(A \cap B))$. Hence $A \cap B$ is $\mathcal{C}_m$-t-set.

Thus the family of $\mathcal{C}_m$-t-set forms an infratopology[7], where an infratopology[7] on a set X is a collection τ of subsets of X having the following properties:

i.) $\emptyset$ and X are in τ.

ii.) The intersection of the elements of any finite sub collection of τ is in τ.

In our next theorem, we provide a decomposition of open set in terms of $\mathcal{C}_m$-pre-open and $\mathcal{C}_m$-t-set.

Theorem 4.11 *Let (X, τ) be a topological space. A subset $S \subseteq X$ is open if and only if S is $\mathcal{C}_m$-pre-open and $\mathcal{C}_m$-t-set.*

Proof. Let $S \subseteq X$ be an open set. Therefore S is $\mathcal{C}_m$-pre-open as well as $\mathcal{C}_m$-t-set.

Conversely, let S be a $\mathcal{C}_m$-pre-open and $\mathcal{C}_m$-t-set. We have $S \subseteq int(\mathcal{C}_m(S)) = int(S) \subseteq S$. Hence S is open.

Now we proceed to provide a decomposition for continuous mappings.

Definition 4.12 Let$f: X \to Y$be a mapping. Then f is said to be

i.) $\mathcal{C}_m$-*pre-continuous* if $f^{-1}(V)$ is $\mathcal{C}_m$-pre-open,

ii.)$\mathcal{C}_m$-*t-continuous* if $f^{-1}(V)$ is a $\mathcal{C}_m$-t-set,

where V is any arbitrary open set in Y.

From the discussion provide above, it follow that $\mathcal{C}_m$-t-continuity does not imply semi-continuity, hence does not imply continuity, α-continuity or A-continuity. We have the following examples in this regard.

Example 4.13 Let$X = \mathbb{R}$, the set of real numbers with the usual topology and$Y = \{a, b\}$with the topology$\mu = \{\emptyset, \{a\}, Y\}$. Let$f: X \to Y$be defined as*:*

$$f(x) \quad = \quad a \qquad where\ A = (5,6) \cap \mathbb{Q}$$

$$= b\ if\ x \notin A$$

Then f is $\mathcal{C}_m$-t-continuous but neither semi-continuous nor continuous.

$\mathcal{C}_m$-t-continuity is independent from B-continuity and A-continuity also. Here are the examples:

Example 4.14 Let$X = \mathbb{R}$, the set of real numbers with the usual topology and$Y = \{a, b\}$with the topology$\mu = \{\emptyset, \{a\}, Y\}$. Let$f: X \to Y$be defined as:

$f(x) = a$ where $A = [5,6) \cup (6,7) \cup \{\pi\}$

$= b$ if $x \notin A$

Then f is $\mathcal{C}_m$-t-continuous but not B-continuous.

Example 4.15 Let $X = \{a, b, c, d\}$, with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $Y = \{x, y\}$ with the topology $\mu = \{\emptyset, \{x\}, Y\}$. Let $f: X \to Y$ be defined as: $f(a) = y, f(b) = f(c) = f(d) = x$. Then f is B-continuous but not $\mathcal{C}_m$-t-continuous.

Similarly,

Example 4.16 Let $X = Y = \{a, b, c, d\}$, with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ on X and topology $\mu = \{\emptyset, \{c\}, \{c, d\}, \{a, b, c\}, Y\}$. Let $f: X \to Y$ be defined as identity map. Then f is A-continuous but not $\mathcal{C}_m$-t-continuous because $\{c, d\}$ is not a $\mathcal{C}_m$-t-set in X.

From **Theorem** 4.11, we have a decomposition of continuity in the following manner:

Theorem 4.17 *A mapping $f: X \to Y$ is continuous if and only if it is both $\mathcal{C}_m$-pre continuous and $\mathcal{C}_m$-t-continuous.*

6. m-REGULARITY

In this section, we further use the concept of m-open sets to define a weaker form of regularity. Some interesting results are obtained here. While Hausdorffness implies m-regularity. Also the product of normal spaces are found to be m-normal.

Definition 5.1 A topological space X is said to be *m-regular* if for each pair consist of a point x and a closed set B not containing x, there exist a disjoint pair of m-open set and an open set, containing x and B respectively.

Since every open set is m-open therefore every regular space is m-regular as well. Converse is however not true. Here is an example.

Example 5.2 Let $X = \mathbb{N}$, the set of natural numbers, equipped with the cofinite topology. Then every subset of X is *m-open.* Then X is a *m-regular* space but not a regular space.

Theorem 5.3 *Let X be a topological space then X is m-regular if and only if given a point $x \in X$ and an open neighbourhood U of x, there exists an m-open neighbourhood V of x such that $x \in V \subseteq cl(V) \subseteq U$.*

Proof. Suppose that X is m-regular space and x and an open neighbourhood U of x are given. Then $B = X \backslash U$, is a closed set not containing x. By the hypothesis, there exists a pair of disjoint m-open set V and open set W, containing x and B respectively, that is, $x \in V$ and $B \subseteq W$. Then $cl(V)$ is disjoint from B and hence $x \in V \subseteq cl(V) \subseteq X \backslash W \subseteq X \backslash B \subseteq U$, that is $cl(V) \subseteq U$.

Conversely, suppose that a point x and a closed set B not containing x are given. Then $U = X \backslash B$, is an open set containing x. Therefore there is a m-open neighbourhood V of x such that $cl(V) \subseteq U$. Then the m-open sets V and the open set $X \backslash cl(V)$, are disjoint sets containing x and B respectively. Thus (X, τ) is m-regular.

Theorem 5.4 *Every Hausdorff space is m-regular.*

Proof. Let X be a Hausdorff space. Let x and B be a pair of a point and a closed set not containing the point x. Then for every $y \in B$, we have $x \neq y$. Therefore by the given hypothesis, there exist disjoint open sets U_y and V_y such that $x \in U_y$, $y \in V_y$ and $U_y \cap V_y = \phi$. Then $\{V_y | y \in B\}$ is an open cover of B and $U = \bigcap_{y \in B} U_y$, is an m-open set containing x such that $U \cap V = \phi$. Therefore X is m-regular.

Our next theorem is on the product of m-regular spaces.

Theorem 5.5 *Product of m-regular spaces is again m-regular.*

Proof. Let $\{X_\alpha\}$ be a family of m-regular spaces. Let $X = \prod_\alpha X_\alpha$. Let $x = \{x_\alpha\}$ be a point of X and U be an open neighbourhood of $x \in X$. We choose a basis element $\prod_\alpha U_\alpha$ about x contained in U. Then $x_\alpha \in U_\alpha$ for each α. As X_α is m-regular, there exists an m-open set V_α in X_α such that $x_\alpha \in V_\alpha \subseteq cl(V_\alpha) \subseteq U_\alpha$. Now, V_α being m-open, we have, $V_\alpha = \bigcap_j V_{\alpha_j}$, where V_{α_j} is open in X_α. If $U_\alpha = X_\alpha$, we simply choose $V_\alpha = X_\alpha$. Then by using the fact [5] that $\bigcap_\beta (\prod_\alpha A_{\alpha,\beta}) = \prod_\alpha (\bigcap_\beta A_{\alpha,\beta})$, we find that $V = \prod_\alpha V_\alpha$ is an m-open set in $\prod_\alpha X_\alpha$. Since $cl(V) = cl(\prod_\alpha V_\alpha) = \prod_\alpha cl(V_\alpha)$, it follows that $x \in V \subseteq cl(V) \subseteq \prod_\alpha U_\alpha \subseteq U$. Hence X is m-regular.

Definition 5.6 A topological space X is said to be *m-normal* if every pair of disjoint closed sets are contained in disjoint *m-open* sets.

One can observe that every normal space is m-normal. But converse is not true in general.

In **Example** 5.2, X is m-normal but not normal.

In our next theorem, we provide that m-regular spaces are m-normal.

Theorem 5.7 *Every m-regular space is m-normal.*

Proof. Let X be m-regular space. Let A and B be disjoint closed subsets of X. Then for each $x \in A$ has an open neighbourhood $X \backslash B$, because $A \subseteq X \backslash B$. By the given hypothesis, there exists an m-open set V_x such that $cl(V_x)$ doesn't intersect B, that is $x \in V_x \subseteq cl(V_x) \subseteq X \backslash B$.

Therefore $\{V_x | x \in A\}$ forms an m-open covering of A. In the same way, $\{U_x | x \in B\}$ is also an m-open covering of B. Thus $V = \bigcup_{a \in A} V_a$ and $U = \bigcup_{b \in B} U_b$ are m-open sets containing A and B respectively.

Now we define $U_i' = U_i \setminus \bigcup_{a \in A, a \neq i} cl(V_a)$ and $V_i' = V_i \setminus \bigcup_{b \in \wedge', b \neq i} cl(U_b)$. Here U_i' and V_i' are m-open sets. Since arbitrary union of closed sets are m-closed, therefore $\bigcup_{a \in A, a \neq i} cl(V_a)$ is m-closed. (Because if A is m-open, then $A = \bigcap_n \{V_n,$ where V_n is open$\}$. Thus $X \setminus A = X \setminus \bigcap_n V_n = \bigcup_n (X \setminus V_n) = \bigcup_n U_n$, where $U_n = X \setminus V_n$, closed sets).Thus $U_i \setminus \bigcup_{a \in A, a \neq i} cl(V_a) = U_i \cap [\bigcup_{a \in A, a \neq i} cl(V_a)]^c$. Here $[\bigcup_{a \in A, a \neq i} cl(V_a)]^c$ is an m-open set in X, therefore $U_i' = U_i \setminus \bigcup_{a \in A, a \neq i} cl(V_a)$ is again an m-open set in X. Similarly, V_i' is m-open. Now, we have $U' = \bigcup U_i'$ and $V' = \bigcup V_i'$ are disjoint m-open covers containing A and B respectively. Therefore X is m-normal space.

Corollary 5.8 Thus from the **Theorem***5.7* and *5.5,* we can say that the product of normal T_1-space is coming out be *m-normal.*

ACKNOWLEDGEMENT

This work is a part of a research work financed by the University Grant Commission (India).

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