A Method for Determination of Shear wave Attenuation Coefficient in Rubbers based on a Piezo-electric Resonator

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Abstract—*Determination of shear wave attenuation coefficient in rubbers has significance in many applications, for example in non-destructive evaluation of bonding of rubber linings, acoustic absorbing/ transparent coatings in air and underwater acoustic devices, etc. Generally an elaborate and expensive acoustic test setup is used for measurement of acoustic properties of rubber and other acoustic materials. In the present work, a piezoelectric resonator method based on bandwidth of a radial mode piezoelectric disc is proposed. Piezoelectric resonators like quartz controlled microbalance (QCM) are well-known for their use in thin film thickness monitor in vacuum coating units based on based on resonance of quartz element. Likewise, quartz oscillator has also been deployed for viscosity determination. The present work describes a radial mode piezoelectric resonator coupled with rubber pads, wherein the radial oscillations excite shear waves into the rubber. The bandwidth of the resonator is related to the penetration depth (attenuation coefficient) of the shear waves. Bandwidth based on data from literature on decay time constant of a damped piezoelectric resonator has been used to compute the shear wave attenuation coefficient in rubbers.A good degree of agreement is observed between the calculated attenuation coefficient and the experimental value from the literature. The proposed methodwould provide a simple solution to measure shear wave attenuation coefficient in rubbers or other visco-elastic media at discrete frequencies.*

Keywords*: Shear wave; attenuation coefficient; piezoelectric resonator; radial mode.*

1. INTRODUCTION

The present work reports on a method based on a piezo-electric resonator for determination of shear wave attenuation coefficient in rubbers. Piezo-electric resonators are piezoelectric solid structures, with or without sandwiched metallic layer, having a definite geometry and size, which can vibrate in a particular mode at a particular frequency. Generally piezoelectric resonators vibrate with high Q, i.e. with a narrow resonant frequency and thus can be used to sense physical effect which changes its frequency. A number of methods based on piezoelectric resonators are known for various applications of sensing in research and industry. Among these mention can be made of Quartz Crystal Microbalances (QCM) which are used for monitoring thickness of thin films in vacuum coating units and their viscoelastic properties [1]. QCMs are based on change of resonant frequency and Q of a quartz crystal vibrating in thickness-shear mode, due to the deposited mass of the coated material. Now-a-days QCMs are used for sensing mass in the realm of pico- to femto-gram. Micro-machined cantilevers, capacitive resonant sensors, etc. have also been successfully used for mass detection in this range. Resonant piezoelectric sensors such as those based on tuning forks have been used for measurement of force [2].

Recently, a low force sensor based on the damped vibration characteristic of a radial mode piezoelectric resonator disc interfaced with a rubber medium has been reported [3]. In the present paper an attempt has been made to extend the physical principle and reported measurements of this work to deduce shear wave attenuation in rubbers. Shear wave attenuation measurement in rubbers is significant for evaluation of bonding and acoustic damping characteristics of rubber linings, acoustic absorbing/ transparent coatings in air and underwater acoustic devices, etc.

2. PIEZOELECTRIC RESONATOR MODEL

The electronic model of the radial mode piezoelectric resonator is described in reference 3. The resonator is based on PZT (lead zirconatetitanate ceramic) material. Thin circular disc of diameter 30 mm and thickness 2 mm, poled along thickness with electrodes on both the sides, has been considered. Under a.c. excitation the disc gets longitudinally deformed along thickness due to the piezoelectric coupling constant d_{33}. Simultaneously, the disc gets radially deformed due to Poisson's ratio. The lowest resonance occurs at the fundamental radial mode having frequency constant N_r=2100 Hz-m. This corresponds to 70 kHz for the diameter of the disc. The next higher mode is the longitudinal thickness resonance having the frequency constant N_t=2020 Hz-m, which corresponds to 1 MHz for the thickness of the disc. At the radial mode resonance, the disc primary vibrates in transverse radial direction with practically insignificant vibration in thickness direction. These vibrations excite shear waves in to the rubber disc which is interfaced with the PZT resonator.

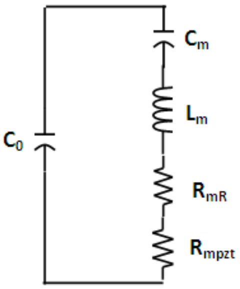

Fig. 1: A series LCR branch in parallel with the blocked capacitance of the PZT. C_m, L_m for motional capacitance and inductance. R_{mR} for motional resistance for radiative load and R_{mpzt} for motional loss in the PZT.

The authors in reference [3] have considered the vibration of the PZT disc at resonance, the equivalent circuit of which is shown in Fig.1, comprising of a series LCR branch in parallel with the blocked capacitance of the PZT disc. C_m, $L_m$represents motional capacitance and inductance. R_{mR} for motional resistance for radiative load and R_{mpzt} for motional loss in the PZT.

3. RESONATOR DECAY TIME CONSTANT

When excited by a pulse, the vibrations of the disc decay after the cut-off excitation voltage with a time constant τ governed by the LCR circuit and has been shown as given by

$$\tau=2.L_m/(R_{mR} + R_{mpzt}) \qquad (1)$$

The loss resistance is predominantly due to radiative losses R_{mR}in the medium, the losses within the PZT being considerably smaller. This is also clear from the difference in the values of mechanical Q of the loaded and unloaded PZT disc.

For static force measurement, the earlier work [3] has considered that the decay time constant τ decreases almost linearly with applied stress and can be treated for force measurement over a small range. No explanation has been given for this response of the decay time constant with applied stress. An explanation of this response of the resonator is given as follows. The rubber-piezoelectric ceramic combination makes a rough interface where rubber makes contact with the ceramic initially at few points. The acoustic coupling (or loading of the piezo-ceramic) is weak at low stress and the pulse decay time constant is large. As the applied stress increases the point contacts become wider, the acoustic coupling into rubber increases and the decay time constant decreases. Clearly, this process shall stabilize when the rough interface becomes smooth interface at large stress. The decay time constant vs applied stress graph should actually saturate at high stress level.

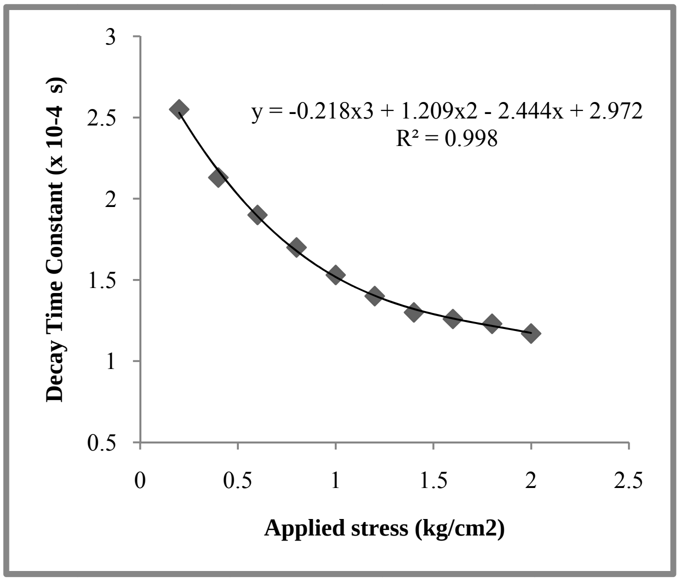

Fig.2: Variation of decay time constant taken from [3] with applied stress between the piezoelectric resonator and the rubber interface.

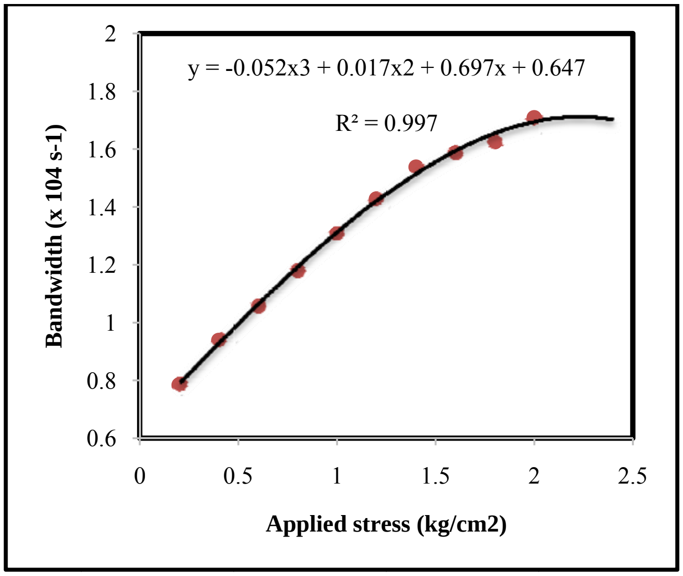

Fig.3: Variation of band width of piezoelectric resonator interfaced with rubber with applied interfacial stress.

A least square polynomial fit of the experimental data of reference [3] with correlation coefficient 0.9976 (Fig.2) shows that the decay time constant saturates at ~ 1.15 10^{-4} s. Under this condition, the decay time constant would be due to the radiated shear wave in the rubber with smooth interface. From this value of τ, the bandwidth of the resonator can be determined as

$$\Delta f\,.\tau/2=1 \qquad (2)$$

The variation of the bandwidth with applied stress also shows a polynomial fit with a high degree of correlation coefficient equal to 0.997. It is to be noted that the decay time constant above is for the case piezo-resonator is sandwiched between two rubber layers. For one rubber layer on one side this decay time constant would be double, i.e. 2.3×10^{-4} s.

4. EVALUATION OF SHEAR WAVE ATTENUATION COEFFICIENT

From the theory of QCM radiating into a viscous medium the half-power bandwidth has been shown [5] to be related to the shear wave penetration depth (attenuation coefficient). The quartz crystal half-power half bandwidth increases to ΔΓ as follows.

$$\Delta\Gamma / f = 1/(\pi . Z_q).[\rho_{liq} .\omega.\delta/2].[1 + 2.h_r^2/\delta^2] \qquad (3)$$

Where Z_q represents the shear acoustic impedance of quartz, ρ_{liq} is the density of the viscous liquid having viscosity η. δ is the shear wave penetration depth given by $(2\eta/\rho_{liq}.\omega)^{1/2}$, which is related to the attenuation coefficient (α) as δ=1/α. h_r is the surface roughness parameter.

Table 1: Material parameters of rubber and PZT disc.

Sl. No.	Parameter	Value
1	Density of rubber	1.7 g cm-3
2	Density of piezoelectric element(PZT)	7.6 g cm-3
3	Radial velocity in PZT	2100 m/s
4	Longitudinal velocity in PZT	4040 m/s
5	Dielectric loss in PZT (tan δE)	0.004
6	Mechanical loss(tan δM)	0.002
7	Mechanical Q of unloaded piezo-resonator	500
8	Mechanical Q of loaded piezo-resonator	16.1

This theory can be applied to the present case where, instead of quartz crystal and viscous medium, a PZT disc and the rubber medium is involved Z_q shall be radial mode acoustic impedance of piezo-ceramic Z_c equal to product of density ($=7.6$ g/cm^3) and radial mode velocity (2.1×10^5 cm/s) of piezo-ceramic. ρ_{liq} shall be the density of rubber ρ_r ($=1.7$ g/cm^3). Under the condition that the piezo-ceramic –rubber sandwich is under large applied stress, the roughness term h_r at the interface can be neglected. The bandwidth Δf of the piezo-ceramic resonator can thus be written as double of ΔΓ as

$$\Delta f/f = 2.[\rho_r .\omega]/(2\pi.Z_c .\alpha)$$

$$\text{Or, } \alpha = 2.\rho_r f^2/(Z_c. \Delta f) \qquad (4)$$

Substituting the values of various quantities in (4), the shear wave attenuation coefficient α at the frequency 70 kHz can be computed as ~ 18 dB per cm. From the literature [6] the value α is seen quite close to this value.

5. CONCLUSION

A method using a piezoelectric resonator is proposed to determine shear attenuation coefficient at specific frequencies in rubbers or similar materials. The model used is based on expression of bandwidth of the radial mode PZT disc radiating shear waves in rubber medium. A close agreement is observed between the computed value and the literature value of shear wave attenuation coefficient at 70 kHz in natural rubber.

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