An Open Loop Method for Time Minimizing Transportation Problem with Mixed Constraints

Swati Agarwal[1], Shambhu Sharma[2]

[1]Research Scholar, Department of Mathematics, Dayalbagh Educational Institute, Dayalbagh, Agra, India
[2]Department of Mathematics, Dayalbagh Educational Institute, Dayalbagh, Agra, India

Abstract: **In real world situations, different sources and destinations impose different restrictions on supply and demand respectively according to their availability and requirement because of the changing economic and environmental conditions. It is also important that the time required for transportation should be minimum. Keeping this in mind, a time minimizing transportation problem with mixed constraints (TMTP-MC) is investigated in which the objective is to minimize the maximum of the total time that is taken to accomplish the requirements of n demand points using the capacities of m supply points. An open loop method has been developed to improve the initial basic feasible solution by shifting the basic cells to other basic cells or non-basic cells having less time. An algorithm is developed to determine an optimal solution for TMTP-MC. The proposed algorithm is found to be conceptually simple and easy to implement. Finally, a numerical illustration has been presented to demonstrate how this algorithm can be used to obtain optimal solution for TMTP-MC.**

1. INTRODUCTION

The time minimizing transportation problem with mixed constraints (TMTP-MC) we consider in this paper is an extension of a classical time minimizing transportation problem (TMTP). TMTP was originally studied by P. L. Hammer [6]. In 1971, Garfinkel and Rao [5] developed a threshold algorithm for the solution of the TMTP. Szwarc [10] précised Hammer's algorithm and proposed a review of the proof of a result in Hammer's paper. He also provided an extensive survey of the solution of TMTP.

The TMTP focuses attention on determining the quantity of a uniform (homogeneous) commodity transported from a source (i.e. factory where the commodity is manufactured) to the respective destination (i.e. warehouse where the commodity is to be distributed) satisfying the given supply and demand limits so that the transportation work is to be finished in minimum time. This problem actually minimizes the maximum of transportation time needed between a supply point and a demand point such that the distribution between the two points is positive. The aim is to minimize the time of transportation which remains independent of the amount of commodity transported. It is assumed that the carriers have enough capacity to carry goods from an origin to a destination in a single trip and, they start simultaneously from their respective origins. The TMTP is come across in connection with transportation of perishable goods whose shelf life is low, with the delivery of emergency supplies, fire supplies, ambulance services or when military equipment are to be delivered from their bases to fronts where prompt delivery is very necessary.

In literature, a good amount of research [2, 5, 7, 8, 9, 10] is available to obtain an optimal solution for the time minimizing transportation problem with equality constraints but the problem with mixed constraints is still untouched by the researchers. The TMTP-MC is widely useful in many real life situations where different sources and destinations impose different restrictions on supply and demand respectively according to their availability and requirement. Some sources want to supply exactly the fixed amount of goods. However, some other sources do not want to supply the amount less than a fixed amount. This case arises for the big factories, companies etc. Rest other sources are not able to supply more than a fixed amount of goods. This case arises for small factories, companies, etc. Similar conditions occur for the destinations as well. Due to the sources and destinations having mixed nature, the problem is said to be a TMTP-MC.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The mathematical model for time minimizing transportation problem is to find t_{ij} and x_{ij} which

$$\text{Minimize } Z = \max_{i,j} \left\{ t_{ij} \,/\, x_{ij} > 0 \right\}$$

Subject to the constraints

$$\begin{cases} \sum_{j=1}^{n} x_{ij} \ge a_i, & i \in \alpha_1 \\ \sum_{j=1}^{n} x_{ij} = a_i, & i \in \alpha_2 \\ \sum_{j=1}^{n} x_{ij} \le a_i, & i \in \alpha_3 \\ \sum_{i=1}^{m} x_{ij} \ge b_j, & j \in \beta_1 \\ \sum_{i=1}^{m} x_{ij} = b_j, & j \in \beta_2 \\ \sum_{i=1}^{m} x_{ij} \le b_j, & j \in \beta_3 \\ x_{ij} \ge 0 \end{cases}$$

$\alpha_i > 0, \forall i \in I$; $\beta_j > 0, \forall j \in J$

$I = \alpha_1 \cup \alpha_2 \cup \alpha_3 = \{1,2.........m\} =$ set of index of supply points.

$J = \beta_1 \cup \beta_2 \cup \beta_3 = \{1,2.........n\} =$ set of index of demand points.

where

$a_i =$ amount of commodity available at ith supply point

$b_j =$ amount of commodity required at jth demand point

$x_{ij} =$ amount of commodity transporting from ith supply point to jth demand point

$m =$ number of supply points

$n =$ number of demand points

$t_{ij} =$ time required for transporting x_{ij} amount of commodity from ith supply point to jth demand point

3. THEORETICAL DEVELOPMENT

To obtain the optimal solution of the problem it is necessary to search an initial basic feasible solution of the problem.

3.1. Initial Basic Feasible Solution

Let us consider a time minimizing transportation problem with mixed constraints (TMTP-MC) involving *m* origins and *n* destinations. Considering, the carriers have enough capacity to transport goods from an origin to a destination in a single trip. It follows that time t_{ij} does not depend on amount of commodity x_{ij}. In 1974, Brigden [3] gave some necessary and sufficient conditions for the existence of a feasible solution for transportation problem with mixed constraints. These conditions are also applicable to TMTP-MC.

Hence, it is always possible to get an initial basic feasible solution to TMTP-MC if it satisfies one of the following conditions:

3.1.1. Necessary and sufficient conditions for the existence of a feasible solution

(i) $\alpha_1 \neq \phi$ and $\beta_1 \neq \phi$

(ii) $\alpha_1 = \phi$ and $\beta_1 \neq \phi$ with

$$\sum_{j\in\beta_1} b_j + \sum_{j\in\beta_2} b_j \le \sum_{i=1}^{m} a_i$$

(iii) $\alpha_1 \neq \phi$ and $\beta_1 = \phi$ with

$$\sum_{i\in\alpha_1} a_i + \sum_{i\in\alpha_2} a_i \le \sum_{j=1}^{n} b_j$$

(iv) $\alpha_1 = \phi$ and $\beta_1 = \phi$ with

$$\sum_{j\in\beta_2} b_j \le \sum_{i=1}^{m} a_i \,,\quad \sum_{i\in\alpha_2} a_i \le \sum_{j=1}^{n} b_j$$

where, ϕ denotes the null set

The initial basic feasible solution can easily be attained by following a straightforward rule. Initiate the method by subtracting the least element of each row from all the elements of the corresponding row of the given transportation table. The element t_{ij} changes to t'_{ij} as follows:

$$t'_{ij} = t_{ij} - \min_j t_{ij} \qquad \forall \quad i = 1,2.........m$$

and then subtract the least element of each column from all the elements of the corresponding column of the transportation table obtained after row operation. The element t'_{ij} changes to t''_{ij} according as

$$t''_{ij} = t'_{ij} - \min_i t'_{ij} \qquad \forall \quad j = 1,2..........n$$

to get the transformed transportation table.

This will give rise to at least one zero in each row and each column. The least element of each row and each column are detected by these zeros. Now, find out the maximum of non-zero row minima and non-zero column minima.

i.e.

$$\max_{i,j}\{R_i, C_j\} = R_r \text{ or } C_s \qquad r \in \{1,2........m\} \text{ and } s \in \{1,2........n\}$$

where,

$$R_i = \min_j (t''_{ij} > 0) \qquad \forall \quad i = 1,2..........m$$

$$C_j = \min_i (t''_{ij} > 0) \qquad \forall \quad j = 1,2..........n$$

If $\max\{R_i, C_j\} = R_r$, then the zero cell of the rth row will be allocated if not, the zero cell of the sth column will be chosen for the allocation.

All allocations are assigned with the help of the table 3.1 which is designed in such a way that only the minimum amount of units is sufficiently assigned to satisfy the whole transportation schedule.

Table 3.1- Assignment of cells

Supply(a_i)	**Demand(b_j)**	**Assign Unit**
$\le$	$\le$	0
=	$\ge$	$\min(a_i, b_j)$
$\ge$	$\ge$	$\min(a_i, b_j)$
$\le$	$\ge$	$\min(a_i, b_j)$
$\ge$	$\le$	$\min(a_i, b_j)$
=	=	$\min(a_i, b_j)$
=	$\le$	$\min(a_i, b_j)$
$\le$	=	$\min(a_i, b_j)$
$\ge$	=	$\min(a_i, b_j)$

Moreover, the row (or column) satisfying the maximum limit of demand and supply will be exhausted. After allocating the first cell, delete the exhausted row or column or both to get the reduced table. In the reduced table, perform the same operation to allocate the second cell. Continuing this process, we will come at the stage where all the demands b_j and supplies a_i corresponding to $i \in \alpha_1 \cup \alpha_2$ and $j \in \beta_1 \cup \beta_2$ are exhausted.

The solution so obtained is our initial basic feasible solution (I.B.F.S.) and the corresponding value of Z is

$Z^{(0)} = \max_{i,j}\{t_{ij} / x_{ij} > 0\} = t_{pq}{}^{(0)}$ where, $Z^{(0)}$ is the maximum time having positive allocations at initial stage. Note that (p, q) cell may or may not be unique.

Remarks

1. In case when $\max\{R_i, C_j\}$ ties, try to choose the row or column with maximum value having one zero cell or break tie arbitrarily.
2. In case when two or more zero cells exist in each row or each column, choose the cell arbitrarily for allotment.
3. The basic cells having zero allocation can be reallocated if necessary. As it is known that, zero allocation exhibits no allocation in the cells.

After obtaining I.B.F.S., we will proceed to its improvement.

3.2. Improvement of the Basic Feasible Solution to Optimal Solution

In this section, we have discussed about how the initial basic feasible solution be improved to get the optimal solution.

Here we want to improve the initial basic feasible solution to get the new improved solution. This can be done by vacating the current basic cell having maximum time. In order to vacate that cell, we can shift the allotment of that cell to its corresponding row or column in the cells having time smaller than that basic cell. The cells whose time is greater than the time in (p, q) cell are of no use, so they can be blocked.

During improvisation, the shifting of allotment can be done partially or wholly i.e., the allotment (x_{ij}) in basic cell with maximum time has been shifted partially or wholly to the other basic cell or non-basic cell having less time as compared to the basic cell having maximum time.

Let the amount to be shifted be $\theta \le x_{pq}$. It is known that in the problem with equality constraints, closed loops were formed to shift the basic cell to a non-basic cell and also helps in determining the non-basic cell eligible to enter into the basis. For making closed loops, a basic cell is targeted to become a non-basic cell. In doing so, θ is to be subtracted from the allocation of that basic cell (partially or wholly) and to make supply and demand balanced, the same amount of θ is to be added to the cell eligible to enter into the basis. Iteratively, the closed loop can be completed. θ is taken as long as the solution remains non-negative such that $x_{ij} - \theta \ge 0$ and $x_{ij} + \theta > 0$. It is also known that only the even number of cells can participate in closed loops.

But in the problems with mixed constraints, it is not required to complete the loop to make supply and demand balanced. To deal with such kind of problems open loops have been introduced which are alike closed loops except the following conditions:

i) basic cell having maximum time (initial cell) is not necessarily a terminal cell,
ii) any number of non-basic cells can enter into the basis besides the solution satisfies the non-negativity,
iii) two open loops can be formed simultaneously from same initial cell,
iv) minimum 2 cells are enough to form an open loop.

The criterion for shifting the basic cell is given in table 3.2

Table 3.2- Shifting criterion of basic cells

	Supply(a_p)	**Demand(b_q)**	**No. of units (θ) shifted to Row/Column**
1.	$= / \ge$	$= / \ge$	
(a)	$\sum_{j=1}^{n} x_{pj} - a_p > 0$	$\sum_{i=1}^{m} x_{iq} - b_q > 0$	$\min\left[\left(\sum_{i=1}^{m} x_{iq} - b_q\right), \left(\sum_{j=1}^{n} x_{pj} - a_p\right)\right]$ units to row/column
(b)	$\sum_{j=1}^{n} x_{pj} - a_p = 0$	$\sum_{i=1}^{m} x_{iq} - b_q > 0$	$\min\left[\left(\sum_{i=1}^{m} x_{iq} - b_q\right), x_{pq}\right]$ units to row

(c)	$\sum_{j=1}^{n} x_{pj} - a_p > 0$	$\sum_{i=1}^{m} x_{iq} - b_q = 0$	$\min\left[x_{pq}, \left(\sum_{j=1}^{n} x_{pj} - a_p\right)\right]$ units to column
(d)	$\sum_{j=1}^{n} x_{pj} - a_p = 0$	$\sum_{i=1}^{m} x_{iq} - b_q = 0$	x_{pq} units to row and column both
2.	$= / \geq$	$\leq$	
(a)	$\sum_{j=1}^{n} x_{pj} - a_p > 0$	_	$\left(\sum_{j=1}^{n} x_{pj} - a_p\right)$ units to row/column
(b)	$\sum_{j=1}^{n} x_{pj} - a_p = 0$	_	x_{pq} units to row
3.	$\leq$	$= / \geq$	
(a)	_	$\sum_{i=1}^{m} x_{iq} - b_q > 0$	$\left(\sum_{i=1}^{m} x_{iq} - b_q\right)$ units to row/column
(b)	_	$\sum_{i=1}^{m} x_{iq} - b_q = 0$	x_{pq} units to column
4.	$\leq$	$\leq$	x_{pq} units to row/column

According to the criteria shown in above table, we can shift the amount from the cell (p, q) to the cell in its row or column without affecting the corresponding demand and supply of the cell. But, whether this shifted amount disturbs the demand and supply of the new cell, where it is shifting? In the context of this question, we have the following cases:

i. If the amount does not disturb the demand and supply of the new cell, then that cell will be the terminal cell.

ii. If the amount disturbs the demand and supply of the new cell, then that amount will pass on to the next new cell in a row or column of the current new cell. It means that we have to search a path starting from (p, q) cell connecting the basic or non-basic cell having less time than that of (p, q) cell and reaching to cell where the shifted amount does not disturb the demand and supply of the last cell of the path. That last cell here is defined as terminal cell.

Now, the question arises here is to how we search the terminal cell.

3.3. Selection of terminal cell

In this section, the criterion of selecting the terminal cells has been discussed.

Here, the set of cells where open loop can terminate in order to improve the basic feasible solution is denoted by T_C. There are two cases arise in selection of the terminal cells which are as follows:

Case 1- If the shifting path goes from $(p,q) \rightarrow (p,a) \rightarrow (b,a) .... (h,i)$ i.e. towards p^{th} row.

$$T_C = \left[(i, j) / t_{ij} < t_{pq}, \quad \forall\ j \in \beta_1\right]$$

In case when β_1 is absent

$$T_C = \left[(i, j) / t_{ij} < t_{pq}, \text{s.t. } b_j - \sum_{i=1}^{m} x_{ij} \geq \theta \ \ \forall\ j \in \beta_3\right]$$

but if $b_j - \sum_{i=1}^{m} x_{ij} < \theta \ \ \forall\ j \in \beta_3$, shift $b_j - \sum_{i=1}^{m} x_{ij}$ units to j^{th} column and search for another $j \in \beta_3$ to shift the remaining $\left[\theta - \left(b_j - \sum_{i=1}^{m} x_{ij}\right)\right]$ units.

Case 2- For the shifting path $(p,q) \rightarrow (r,q) \rightarrow (r,t) .... (o,z)$ i.e. towards q^{th} column.

$$T_C = \left[(i, j) / t_{ij} < t_{pq}, \quad \forall\ i \in \alpha_1\right]$$

In case when α_1 is absent

$$T_C = \left[(i, j) / t_{ij} < t_{pq}, \text{s.t. } a_i - \sum_{j=1}^{n} x_{ij} \geq \theta \ \ \forall\ i \in \alpha_3\right]$$

but if

$a_i - \sum_{i=1}^{m} x_{ij} < \theta \ \forall i \in \alpha_3$, shift $a_i - \sum_{i=1}^{m} x_{ij}$ units to i^{th} row

and search for another $i \in \alpha_3$ to shift the remaining $\left[\theta - \left(a_i - \sum_{j=1}^{n} x_{ij}\right)\right]$ units.

Remark- The terminal cells will never exist in i^{th} row/ j^{th} column $\forall \ i \in \alpha_2$ and $j \in \beta_2$.

After searching for the terminal cell, we shift the amount to the terminal cell to get the improved solution and the corresponding value of Z as $Z^{(1)}$. Repeating the process iteratively, we can obtain $Z^{(2)}$, $Z^{(3)}$ …and so on.

Now question comes here, at what extent we can improve the solution. For this, we need an optimality criterion, which is given below:

3.4. Optimality condition

The solution $t_{pq}^{(n)} = \min_{i,j}\{\max t_{ij} / x_{ij} > 0\} = Z^{(n)}$ is said to be optimal to the Time Minimizing Transportation Problem with Mixed Constraints (TMTP-MC), if one of the given conditions holds:

i.) $\underset{j \neq q}{t_{pj}} \geq t_{pq}$ and $\underset{i \neq p}{t_{iq}} \geq t_{pq}$

ii) $\underset{j \neq q}{t_{pj}} < t_{pq}$,for some $j = 1,2.....n$ and $\underset{i \neq p}{t_{iq}} \geq t_{pq}$,for the shifting path $(p,q) \rightarrow (p,a) \rightarrow (b,a) ....(h,i)$ $s.t.\ \sum_{i=1}^{m} x_{ia} - b_a = 0$ with $x_{ba} < \theta$ where, $a \in \beta_2 \cup \beta_3$ or, $(h,i) \notin T_C$

and/or, $(p,q) \rightarrow (r,q) \rightarrow (r,t) ....(o,z)$

iii.) $\underset{j \neq q}{t_{pj}} \geq t_{pq}$ and $\underset{i \neq p}{t_{iq}} < t_{pq}$,for some $i = 1,2.....m$,for the shifting path $(p,q) \rightarrow (p,a) \rightarrow (b,a) ....(h,i)$.

and/or, $(p,q) \rightarrow (r,q) \rightarrow (r,t) ....(o,z)$

iv.) $s.t.\ \sum_{j=1}^{n} x_{rj} - a_r = 0$ with $x_{rt} < \theta$ where, $r \in \alpha_2 \cup \alpha_3$ or, $(o,z) \notin T_C$.

$\underset{j \neq q}{t_{pj}} < t_{pq}$,for some $j = 1,2.....n$ and $\underset{i \neq p}{t_{iq}} < t_{pq}$,for some $i = 1,2.....m$,for the shifting path

$(p,q) \rightarrow (p,a) \rightarrow (b,a) ....(h,i)$ $s.t.\ \sum_{i=1}^{m} x_{ia} - b_a = 0$ with $x_{ba} < \theta$ where, $a \in \beta_2 \cup \beta_3$ or, $(h,i) \notin T_C$.

and/or, $(p,q) \rightarrow (r,q) \rightarrow (r,t) ....(o,z)$ $s.t.\ \sum_{j=1}^{n} x_{rj} - a_r = 0$ with $x_{rt} < \theta$ where, $r \in \alpha_2 \cup \alpha_3$ or, $(o,z) \notin T_C$.

Remark

An open loop algorithm terminates in finite number of steps because the minimum time attained at each iteration corresponds to an extreme point of the set of feasible solutions of TMTP-MC and there is a finite number of extreme points of the feasible set.

Summarizing all the results obtained above, an algorithm to solve TMTP-MC has been developed which is presented in section 4.

4. ALGORITHM

An algorithm of the TMTP-MC to enumerate an optimal solution consists of the following steps:

Step 0: Construct the transportation table for the given problem.

Step 1: Subtract the least element of each row from all the elements of the corresponding row of the given transportation table.

Step 2: Subtract the least element of each column from all the elements of the corresponding column of the transportation table obtained in step 1.

Step 3: Identify the row (or column) with the maximum of non-zero row minima and column minima and allocate the zero cell of that row (or column) using the Table 3.1.

Step 4: Row (or column) satisfying the maximum limit of demand and supply point will be eliminated. Now, go to step 1 to allocate the second cell. Continuing in this way, an initial basic feasible solution is obtained.

Step 5: Starting from the basic cell having maximum time, draw an open-loop to improve the solution by assigning negative sign (-) at the starting point and alternatively assigning positive (+) and negative (-) signs at the nodes of the open-loop.

Step 6: Add the value of θ (the value of θ can be considered with the help of table 3.2.) to all the cells of the open loop marked with positive (+) sign and subtract it from those cells marked with negative (-) sign. In this way, the current solution becomes improved.

Step 7: Repeat steps 5 to 7 until an optimality condition is achieved.

5. NUMERICAL ILLUSTRATION

In this section, we present a detailed example for an illustration of the above algorithm.

Example: Consider a time minimizing transportation problem with three sources O_1, O_2 and O_3 and five destinations D_1, D_2, D_3, D_4 and D_5.

Table- 5.1

	D_1	D_2	D_3	D_4	D_5	
O_1	8	6	8	6	10	≤ 80
O_2	9	8	9	6	7	≥ 120
O_3	11	10	8	13	9	= 140
	= 40	≥ 40	= 60	≤ 80	≥ 80	

Step 1-2: Subtracting the row minima and column minima from all the elements of the corresponding row and corresponding column respectively. We get the following transportation table 5.2.

Table- 5.2

	D_1	D_2	D_3	D_4	D_5	
O_1	0	0	2	0	3	≤ 80
O_2	1	2	3	0	0	≥ 120
O_3	1	2	0	5	0	= 140
	= 40	≥ 40	= 60	≤ 80	≥ 80	

Step 3: The maximum of non-zero row minima and non-zero column minima is 5 corresponding to the column 4 as shown in table 5.4. Now allocate the zero cell of 4th column with 80 units. We get the table as given below:

Table- 5.3

	D_1	D_2	D_3	D_4	D_5		Row Min.
O_1	0	0	2	0	3	≤ 80	2
O_2	1	2	3	80 / 0	0	≥ ~~120~~ 40	1
O_3	1	2	0	5	0	= 140	1
	= 40	≥ 40	= 60	≤ ~~80~~ 0	≥ 80		
Col. Min.	1	2	2	5	3		

Step 4: 4th column of the above table satisfies the maximum limit of demand (b_4) .Therefore, it will cancelled out and we will continue to allocate the other cells.

Table- 5.4

	D_1	D_2	D_3	D_5		Row Min.
O_1	0	0	2	3	≤ 80	2
O_2	1	2	3	40 / 0	≥ ~~40~~ 0	1
O_3	1	2	0	0	= 140	1
	= 40	≥ 40	= 60	≥ ~~80~~ 40		
Col. Min.	1	2	2	3		

Proceeding in a similar way, we have the following allocations in the given time minimizing transportation table:

Table-5.5

	D_1	D_2	D_3	D_4	D_5	
O_1	8	6	8	6	10	≤ 80
O_2	9	40 / 8	9	80 / 6	40 / 7	≥ 120
O_3	40 / 11	10	60 / 8	13	40 / 9	= 140
	= 40	≥ 40	= 60	≤ 80	≥ 80	

Here, the Initial Basic Feasible Solution has obtained with $Z^{(0)} = 11$: **Step 5-6:** Starting from the (3, 1) cell having maximum time 11, an open loop is formed to vacate that cell. To satisfy the corresponding demand and supply of that cell, two open loops have formed and the cell (3, 4) can be blocked as shown in the following table.

Table-5.6

	D_1	D_2	D_3	D_4	D_5	
O_1	8	6	8	6	10	≤ 80
O_2	9	40 / 8	9	80 / 6	40 / 7	≥ 120
O_3	40 / 11	10	60 / 8	13	40 / 9	= 140
	= 40	≥ 40	= 60	≤ 80	≥ 80	

Now, shifting the allotment of cell (3, 1) to cell (2, 1) and cell (3, 5) as directed in table 5.6 we will get the new improved time i.e., $Z^{(1)} = 9$ with $\sum x_{pq} = 120$

Table-5.7

	D_1	D_2	D_3	D_4	D_5	
O_1	8	6	8	6	10	≤ 80
O_2	40 / 9	40 / 8	9	80 / 6	40 / 7	≥ 120
O_3	11	10	60 / 8	13	80 / 9	= 140
	= 40	≥ 40	= 60	≤ 80	≥ 80	

In the above table, the allotment of cell (2, 1) has been shifted to the cell (1, 1) for further improvement. Here, we get $Z^{(2)} = 9$ with $\sum x_{pq} = 80$. The allotment of cell (3, 5) cannot be shifted to its corresponding row and column, so the solution is optimal by the (iv) optimality criteria.

Table-5.8

	D_1	D_2	D_3	D_4	D_5	
O_1	40 / 8	6	8	6	10	≤ 80
O_2	9	40 / 8	9	80 / 6	40 / 7	≥ 120
O_3	11	10	60 / 8	13	80 / 9	= 140
	= 40	≥ 40	= 60	≤ 80	≥ 80	

Hence, we get the optimal solution for the given TMTP-MC and the corresponding value of $Z^{(2)} = 9$ with $x_{21} = 40, x_{22} = 40, x_{24} = 80, x_{25} = 40,\ x_{33} = 60$ and $x_{35} = 80$

6. CONCLUSION

We have developed a simple algorithm for solving a TMTP-MC. The method presented and discussed above gives us the optimal value in finite number of iterations. The proposed algorithm is easy to understand and apply. It can serve the managers by providing the solution to a variety of distribution problems with mixed constraints. Carriers with limited capacity to carry goods in a single trip also reflect more practical situation in TMTP-MC and are needed for further study.

REFERENCES

[1] Bansal, S., and Puri, M.C., "A min max problem", *Zeitschrift für Operations Research*, 24, 1980, pp. 191-200.

[2] Bhatia, H.L., Swarup, K., and Puri, M.C., "A procedure for time minimization transportation problem", *Indian Journal of Pure and Applied Mathematics*, 8, 1977, pp. 920-929.

[3] Brigden, M.E.B., "A varient of the transportation problem in which the constraints are of mixed type", *Operational Research Quarterly*, 125, 3, 1974, pp. 437-445.

[4] Dantzig, G.B., *Linear programming and extensions*, Princeton University Press, Princeton, NJ.

[5] Garfinkel, R.S., and Rao, M.R., "The bottleneck transportation problem", *Naval Research Logistics Quarterly*, 18, 1971, pp. 465-472.

[6] Hammer, P.L., "Time minimizing transportation problem", *Naval Research Logistics Quarterly*, 16, 1969, pp. 345-357.

[7] Hitchcock, F.L., "The distribution of a product from several sources to numerous localities", *Journal of Mathematics and Physics*, 20, 1941, pp. 224-230.

[8] Pandian, P., and Natarajan, G., "A new method for solving bottleneck-cost transportation problems", *International Mathematical Forum*, 10, 2011, pp. 451-460.

[9] Sharma, J.K., and Swarup, K., "Time minimizing transportation problems", in *Proceedings of Indian Academy of Sciences-Mathematical Sciences*, 86, 1977, pp. 513-518.

[10] Szwarc, W., "Some remarks on the time transportation problem", *Naval Research Logistics Quarterly*, 18, 1971, pp. 473-485.