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Sacred Geometry Mathematical and Geometric Applications in the Ancient Architecture of India

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Source: Author's Own Photo

1. INTRODUCTION

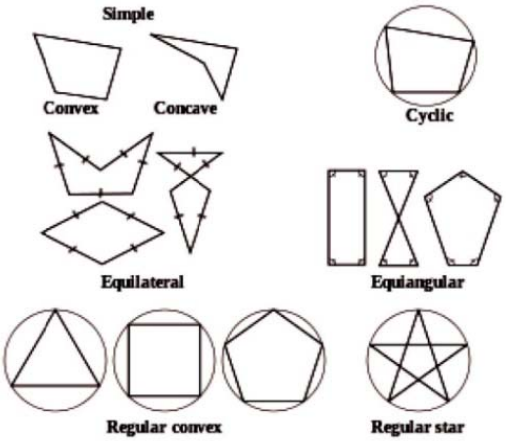
Most people know about the beauty and majesty of the Taj Mahal, one of the architectural wonders of the world. Fewer people know about the mathematics and geometric principles that underlie not just this structure, but many of the ancient Mughal and Hindu structures in India.

The purpose of this article is to shed more light on the symbolic and stylistic use of geometry in India's ancient tombs, temples and mosques. This is by no means an exhaustive review of India's architecture. Rather, it's meant as a brief overview to enhance appreciation for India's built heritage for students of mathematics, architecture, and history alike.

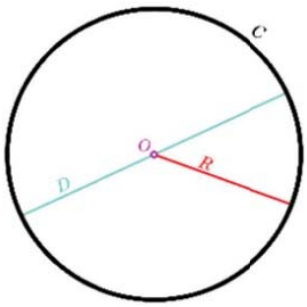
Mughal Architecture

Geometry within Mughal Islamic Architecture in India

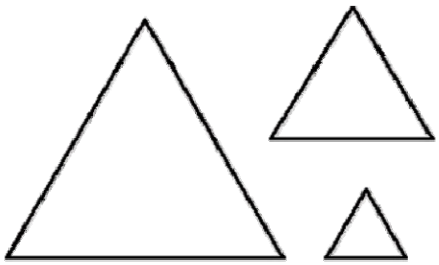
Geometry is one of the most important elements of Islamic Art. A close observation of Islamic architecture shows that geometric shapes like stars, triangles, circles, squares, hexagons and different types of polygons are repeatedly used. From repetition of these basic shapes, more complicated patterns emerge. Within Islamic architecture, complex geometric designs create an impression of unending repetition, which serves to evoke and remind people of the infinite nature of Allah. (The Metropolitan Museum of Art, 2001)



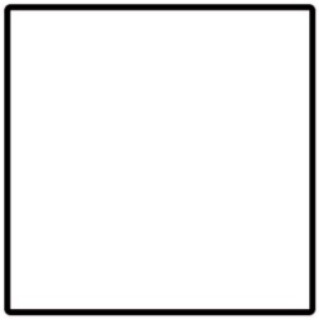
Various polygons used in Islamic patterns



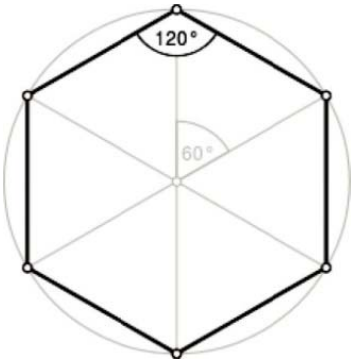
Symbolically, the circle depicts Allah's infinite nature. Almost all Islamic patterns begin at the centre of a circle. This also symbolises being one with God (Allah).



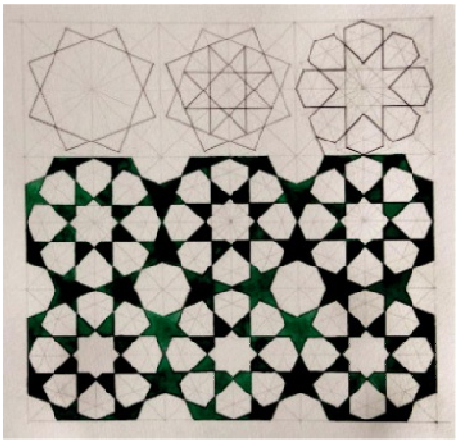
The Triangle symbolises human existence and the principles of humanity.



The square is a symbol of the material world and physical experience.



The Hexagon is a symbol of heaven.



The Star is a symbol of the spread of Islam.

Image Source: Guneet Khurana. *Geometry of Islamic Architecture*. February 18, 2013.

2. TYPES OF ISLAMIC GEOMETRIC PATTERNS

Islamic Geometric Patterns (IGP) are mostly based on polygons such as Hexagons and Octagons. The Star Polygon is the fundamental element of IGP. Patterns whose main elements are from hexagon are classified as 6- point geometrical patterns; a star is called a 6- point star. Accordingly, patterns are labeled as 8-, 10-, 12-... point geometrical patterns. (Adbullahi & Bin Embi, 2013)

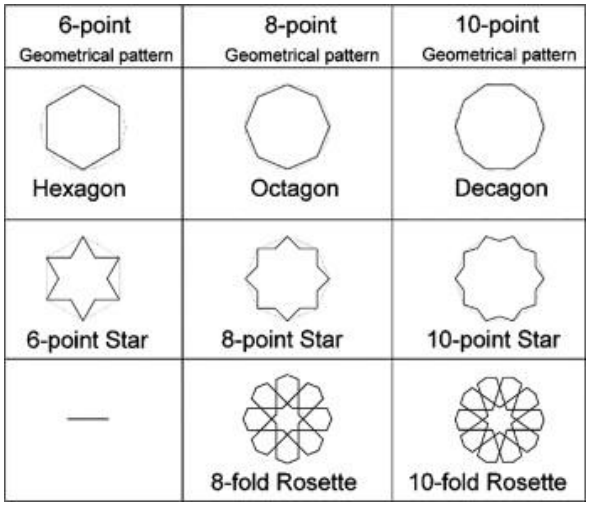


Image Source: Yahya Abdullahi and Mohamed Rashid Bin Embi. "Evolution of Islamic Geometric Patterns," in *Frontiers of Architectural Research*.

3. EXAMPLES OF GEOMETRY IN MUGHAL ARCHITECTURE

Humayun's Tomb

Humayun's Tomb in Delhi (1565 AD) is one of the most iconic examples of Mughal Architecture in India. Examining the various geometric patterns employed, one can see examples of 6-point and 8-point patterns in the marble flooring, window grills, and balcony railings of the Tomb.

The aerial view of Humayun's Tomb shows the extensive use of geometry. The base of the tomb is a cube with arches made up of rectangles and triangles.

Image Source: "Humayun's Tomb" *Cultural India.* http://www.culturalindia.net/monuments/humayun-tomb.html

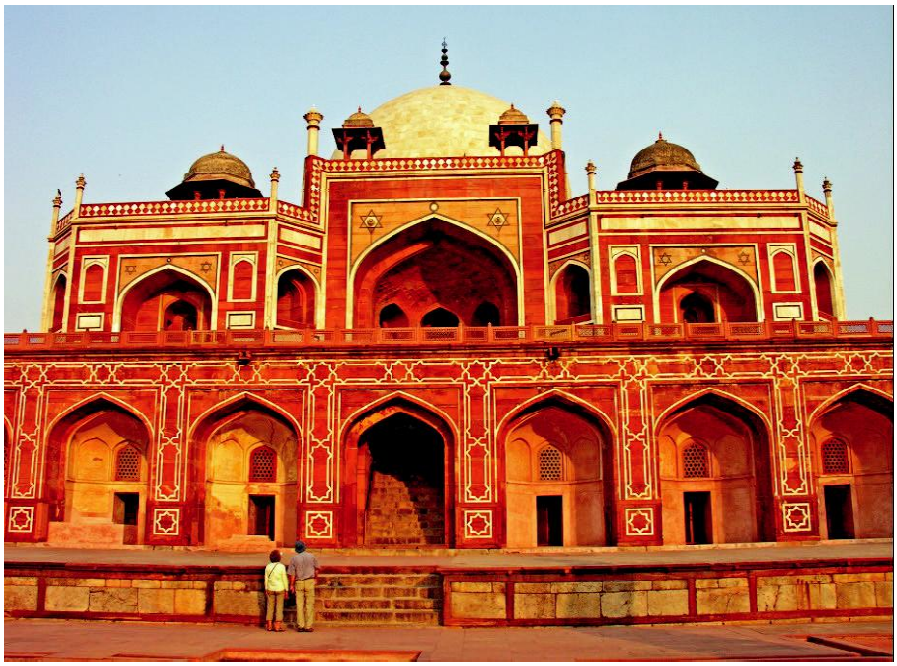


Image Source: “Humayun’s Tomb,” *TripHobo*. https://www.triphobo.com/places/delhi-india/humayun-tomb

The use of geometrical shapes can be seen in not only the tomb but also in the lay out of the gardens. The above picture shows an octagon shaped pond.

Image Source: “Humayun’s Tomb” *Cultural India.* http://www.culturalindia.net/monuments/humayun-tomb.html

The Red Fort

The 6-point and 8-point patterns are also repeated in the Red Fort of Agra (1546 AD). One can also see the occasional use of 10-point and 12-point examples in the Red Fort (Adbullahi

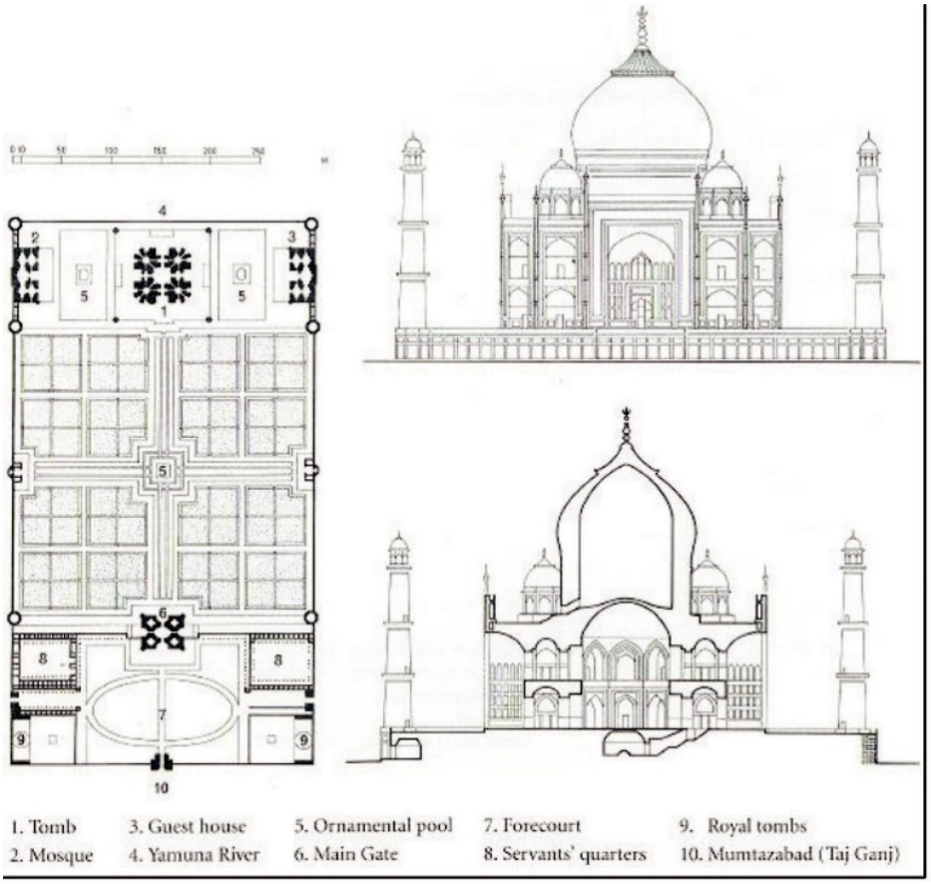
Bin Embi, 2013). Mughal architecture in India used red sandstone, white marble and polychromatic titles as the main cladding and decorative materials. Great effort was exerted to create accurate and perfect proportions of pattern shapes and angles. (Asher, 1992) In addition to these materials, IGP are one of the key decorative elements of both secular and religious buildings in Islamic architecture. Geometric designs were used in an intricate manner to give birth to various patterns to decorate the floors, ceilings, walls, windows and gardens.

The Red Fort in Agra. Image source: Mahesh Bhanupanth. https://commons.wikimedia.org/wiki/File:Red-Fort,Delhi.JPG

Stonework Detail from various Mughal Architecture sites (From left: Humayun Tomb in Delhi; Red Fort in Agra; Friday Mosque of Fatehpur-Sikri; Etimad-ud-Daulah tomb; Lahore Fort in Pakistan.)

Image Source: Yahya Abdullahi and Mohamed Rashid Bin Embi. "Evolution of Islamic Geometric Patterns," in *Frontiers of Architectural Research*.

Taj Mahal



Perhaps the best-recognised example of Mughal architecture is the Taj Mahal. The Taj Mahal was constructed in 1632 AD in Agra, and it is a perfect example of the use of symmetry and geometry, both inside and outside. Throughout the monument, one can see 6-point, 8-point, 10-point,12-point, 16- point and other complex patterns.

From a bird's eye view, one can see that the tomb is in the shape of a cube with chambered corners to give it an octagonal cross section. The octagonal structure of the tomb symbolises the physical world and the path to heaven. The Taj is built with such accurate mathematical calculation that the four minarets that surround the structure would fall outwards in the event of a serious earthquake, so as not to damage the main tomb.

Floor Plan of The Taj Mahal. Image Source: http://islamicarchitectureinindia.weebly.com/taj-mahal.html

One of the most remarkable parts of the Taj Mahal is its enormous onion-shaped dome, which is located at the exact centre of the structure, and is flanked by four smaller domes, one on each corner. At 115 ft, the marble cylindrical dome is topped with a lotus design - one of the most stunning features of the entire structure.

Image Source :http://www.taj-mahal.net/newtaj/textMM/images/domeexterior7.jpg

Hindu Temple Architecture

Overview of Hindu Temple Architecture

Starting in the 1st century CE, there arose a new style of bhakti Hinduism, in which devotees worshipped central deities. This practice of worship necessitated temples, whose formal style was codified over time. In particular, this temple architecture relied heavily on the use of harmonious geometry, which, interestingly, is very similar to fractal geometry. Hindu temples offer some of the best examples of fractal buildings, which were constructed long before the birth of fractal theory and manifested the religious cosmic visions (Adane & Dutta, 2014)

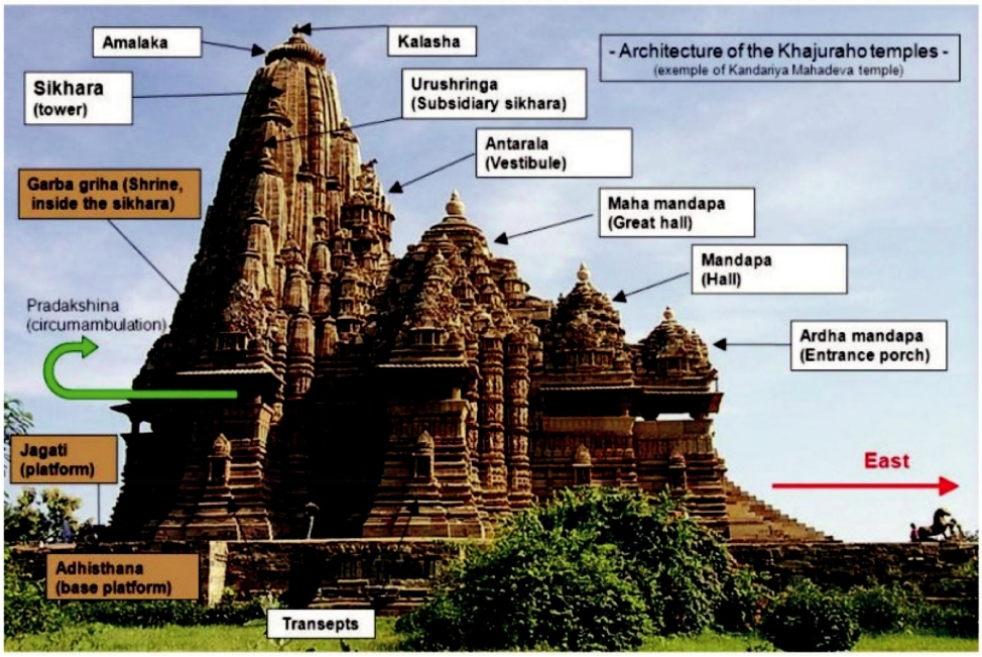


Image Source: "Hindu Architecture" in Ancient History Encyclopedia. https://www.ancient.eu/Hindu_Architecture/

Overview of Fractal Geometry

Fractal geometry is a field of mathematics identified in the 1970s by Benoit Mandelbrot. The process by which shapes are made in fractal geometry is very simple, yet contrasts starkly with classical geometry. While classical geometry uses formulas to define a shape, fractal geometry uses iteration. A fractal is a never-ending pattern, created by repeating a similar process over and over in an ongoing feedback loop. (Physorg.com, 2011)

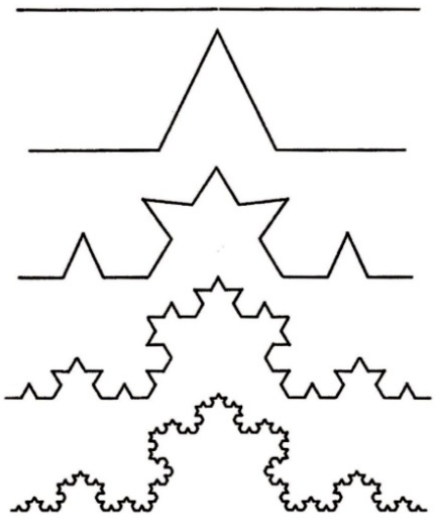
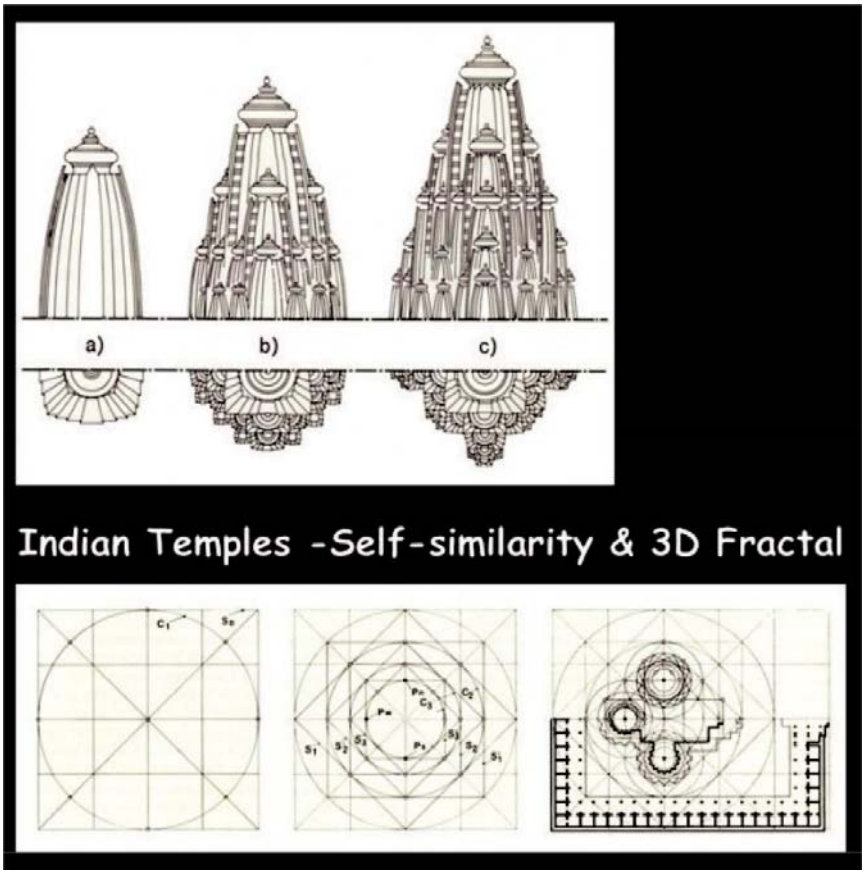


Illustration of self-repetition in fractal geometry. Image Source: Bovill, Carl. Fractal geometry in architecture and design. 1996

Symbolism of Fractal Geometry in Hindu temple architecture



George Michell, author of *The Hindu Temple* explains, "The architecture of the Hindu temple symbolically represents the quest for moksha - ultimate spiritual liberation, the realisation of oneness by setting out to dissolve the boundaries between man and the divine." (Michell, 1977) Temples are a symbol of the cosmos and their form symbolically represents the cosmos.(Trivedi, 1993)

Fractal geometry, which represents "views of the cosmos to be holonomic and self-similar in nature" also has connections to cosmology. For example, in 1987 the Italian physicist Luciano Pietronero argued that the Universe shows "a definite fractal aspect over a fairly wide range of scale" based on correlations of galaxies and clusters, their spatial distribution and average mass density. (Dataisnature, 2015)

Within Hindu temples, one can see uses of fractal geometry, namely through the repetition of geometrical towers, surrounded by mirror images of smaller, similar geometrical towers at various levels. This repetition creates the illusion of an object repeating itself endlessly, towards a proposed infinity.

Image Source: "The Fractal Structure of Hindu Temples" https://fractalenlightenment.com/14556/fractals/the-fractal-structure-of-hindu-temples

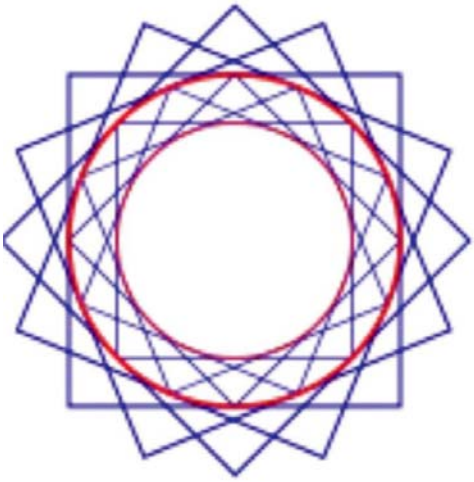
Examples of Fractal Geometry in Hindu Temple Architecture *Kandariya Mahadev Temple, Khajuraho*

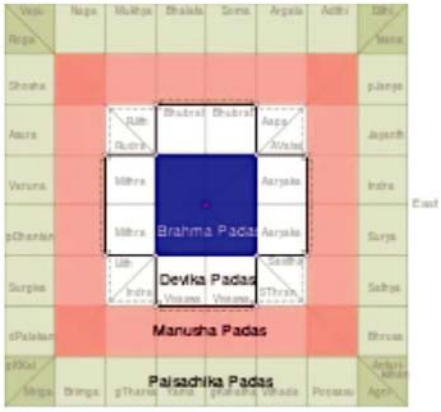
The Khajuraho Temples constructed by the Chandela dynasty between 900 AD and 1130 AD show incredible symmetry and use of fractal geometry. Following the traditional grid geometrical design called vastu -purusha-mandala, the temples in the complex are laid out in a symmetrical, concentrically layered, self-repeating structure around the core of the temple, called the garbhagriya. The shikhara, or spire of the temple, rises above the garbhagriya. This structure and symmetry convey core beliefs and mathematical principles.

Within the Khajuraho temple complex, the Kandariya Mahadev temple is one of the best examples of recursive temple architecture in India. The Shikhara (rising towers) are said to mimic the forms of mountains, which are themselves self-similar. (Dataisnature, 2015) The pillared halls near the entrances (mandapas) are also arranged by principles of symmetry, grids and mathematical precision.

Kandariya Mahadev Temple in Madhya Pradesh Image Source: http://www.dataisnature.com/?p=2138

Khajuraho temples use the 8×8 (64) Vastupurusamandala Manduka grid layout plan (right) found in Hindu temples. Above the temple's brahma padas is a Sikhara (Vimana or Spire) that rises symmetrically above the central core, typically in a circles and turning-squares concentric layering design that flows from one to the other as it rises towards the sky.(Munshi 2015)





Manduka Mandala - Hindu Temple 64 padas

Image Source: Sayan Munshi, "Architecture of the Khajuraho Temples". https://curatorhall.wordpress.com/2015/05/10/architecture-of-the-khajuraho-temples/

The Sun Temple, Modhera

The Sun Temple at Modhera (1026-27 AD) is one of the earliest Solanki temples of Gujarat. In its architectural elements, particularly its reservoir, it show elements of self-repeating fractal geometry. Measuring 176 feet by 120 feet, the reservoir (kunda) is paved with stones all around. There are four terraces with recessed steps to descend to the bottom of the tank. The rectangular and square shape of these steps, combined with their repeated pattern yields a fractal effect, meant to reflect the process of the universe and its energy. (Jain-Neubauer, 1981)

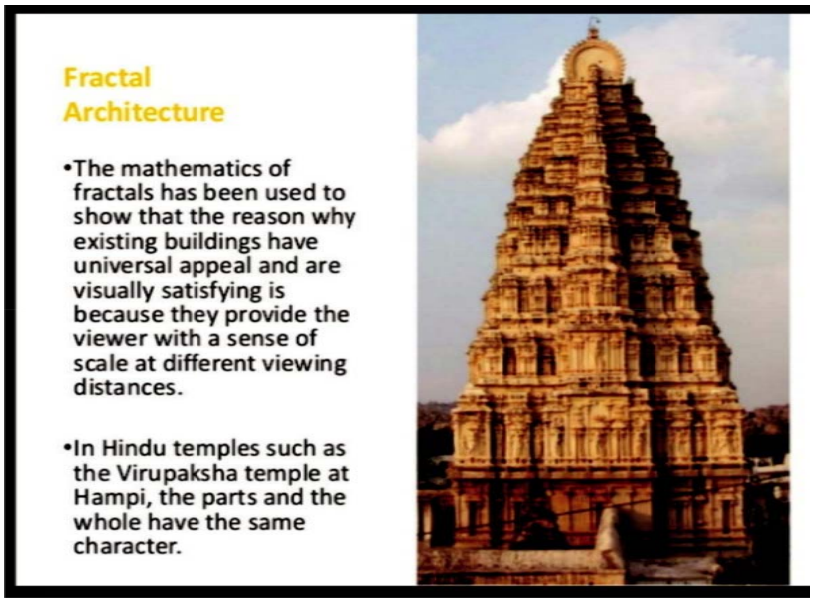
The Reservoir at the Sun Temple at Modhera

Image Source: Tanisha Dutta & Vinayak S. Adane "Symbolism in Hindu Temple Architecture & Fractal Geometry" https://www.ijsr.net/archive/v3i12/U1VCMTQzMjI=.pdf

Image Source: Prayash Giria - https://commons.wikimedia.org/w/index.php?curid=28536799

Virupaksha Temple, Hampi

The Virupaksha Temple at Hampi (740 AD) represents the pinnacle of Dravidian temple architecture. The temple is built on five fully developed moldings of a high plinth. A sense of geometric symmetry is created through the outer faces of the sanctum walls, which are divided into two intermediate projections and two corner projections with four recesses in between. Similarly, within the *mandapa*, the walls on either side of the northern, eastern and southern regions are divided into two projections and two recesses. The superstructure over the sanctum is three storeys tall, and repeats many elements of the parapet and walls beneath. (Archaeological Survey of India, 2011)



In the architecture of the Virupaksha Temple, the use of fractal geometry means that both the parts of the tower, and the tower as a whole have the same character. This explains the universal appeal of the structure, as it provides viewers with a sense of scale, even when viewing it from different distances.

Tower of the Virupaksha Temple at Hampi. Image Source: http://templenet.com/Karnataka/virupa.jpg

Aerial View of Virupaksha Temple at Hampi

Image Source: https://image.slidesharecdn.com/mathematicsandart-150505062338-conversion-gate02/95/mathematics-and-art-31-638.jpg?cb=1501435168

4. CONCLUSION

Whether in the form of traditional or fractal geometry, applications of mathematics and geometry play a key role in not only giving India's ancient architecture its characteristic look, but also expressing significant symbolic aspects. Understanding the basic geometry and corresponding symbolism underlying these structures yields a deeper appreciation for India's history and heritage, through it sacred spaces.

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